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On Strongly T #g*-Space, Strongly g∗ C-Homeomorphisms and Strongly g∗-Connectedness in Topological Spaces

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Abstract

In this paper, we introduce the space called On Strongly $T^{\#g}$-space and homeomorphism called Strongly g^* C-homeomorphisms and Strongly g^*-connectedness in topological spaces and studied some of their properties.

Keywords:— *Strongly $S_{\#}$-G^*-closed set.*

2000 Mathematics Subject Classification: 57N40.

I. Introduction

The notion homeomorphism plays a very important role in topology. By definition, a homeomorphism between two topological spaces X and Y is a bijective map $f : X \rightarrow Y$ when both f and f^{-1} are continuous. It is well known that as Janich [1] says correctly: homeomorphisms play the same role in topology that linear isomorphisms play in linear algebra, or that biholomorphic maps play in function theory, or group isomorphisms in group theory, or isometries in Riemannian geometry. In the course of generalizations of the notion of homeomorphism, Makiet al. [2] introduced g-homeomorphisms and gc-homeomorphisms in topological spaces. Recently, Devi et al. [3] studied semi-generalized homeomorphisms and generalized semi-homeomorphisms.

A space which has an associated family of subsets that constitute a topology. The relationships between members of the space are mathematically analogous to those between points in ordinary two-and three-dimensional space. In topology and related branches of mathematics, a topological space may be defined as a set of points, along with a set of neighbourhoods for each point, satisfying a set of axioms relating points and neighbourhoods. The definition of a topological space relies only upon set theory and is the most general notion of a mathematical space that allows for the definition of concepts such as continuity, connectedness, and convergence. Other spaces, such as manifolds and metric spaces, are specializations of topological spaces with extra structures or constraints. Being so general, topological spaces are a central unifying notion and appear in virtually every branch of modern mathematics. The branch of mathematics that studies topological spaces in their own right is called point-set topology or general topology. Andrijevic [4] introduced the notion of b-open sets in a topological space and obtained their various properties. El-Etik [5] introduced the same concept in the name of γ-open sets. El-Etik also introduced the concept of γ-continuous (*b*-continuous) functions with the aid of *b*-

open sets. In 2004, Ekici and Caldas [6] introduced the notion of slightly γ-continuity (slightly *b*-continuity) which is a weakened form of *b*-continuity. In their paper, the authors have studied basic properties and preservation theorems of slightly *b*-continuous functions. The relationships of slightly *b*-continuity with other weaker forms of continuity have also been studied. The concept of generalized closed sets(briefly *g*-closed) in topological spaces was introduced by Levine [7] and a class of topological spaces called $T_{½}$ spaces.

Arya and Nour [8], Bhattacharya and Lahiri [9], Levine [10], Mashhour [11], Njastad [12] and Andrijevic([36], [4]) introduced and investigated generalized semi-open sets, semi generalized open sets, generalized open sets, semi-open sets, pre-open sets and α-open sets, semi pre-open sets and *b*-open sets which are some of the weak forms of open sets and the complements of these sets are called the same types of closed sets.

Tong ([14], [15]) has introduced *A*-sets, *B*-sets and *t*-sets. *A*-sets and *B*-sets are also weak forms of open sets whereas *t*-sets is a weak form of a closed sets. Ganster and Reilly [16] have introduced locally closed sets, which are weaker than both open and closed sets. Cameron [17] has introduced regular semi-open sets which are weaker than regular open sets.

Generalized open sets play a very important role in General Topology and they are now the research topics of many topologists worldwide. Indeed a significant theme in General Topology and Real analysis concerns the variously modified forms of continuity, separation axioms etc. by utilizing generalized open sets. For a subset A of a topological space (X, τ), $Cl(A)$ and $Int(A)$ denote the closure of A and the interior of A, respectively. Wilansky [18] has introduced the concept of *U S* spaces. Aull [19] studied some separation axioms between the T_1 and T_2 spaces, namely, S_1 and S_2. Next, S. P. Arya et al. [20] have introduced and studied the concept of semi-*U S* spaces in the year 1982 and also they made study of *s*-convergence, sequentially semi-closed sets, sequentially s-compact notions. G. B. Navlagi studied *P*-Normal Almost-*P*-Normal and Mildly-*P*-Normal spaces. Closedness are basic concept for the study and investigation in general topological spaces. This concept has been generalized and studied by many authors from different points of views. O.Njastad [12] introduced and defined an α-open and α-closed set. After the works of O.Njastad on α-open sets, various mathematicians turned their attention to the generalizations of various concepts in topology by considering semi-open, α-open sets. The concept of g-closed [7], *s*-open [10] and α-open sets has a significant role in the generalization of continuity in topological spaces. The modified form of these sets and generalized continuity were further developed by many mathematicians ([21], [22], [23], [24], [11]). Many authors have tried to weaken the condition closed in this theorem. In 1978, Long and Herrington [25] used almost closedness due to Singal [26]. Malghan [11] introduced the concept of generalized closed maps in topological spaces. Devi [28] introduced and studied *sg*-closed maps and *gs*-closed maps. *wg*-closed maps and *rwg*-closed maps were introduced and studied by Nagavani [29]. Regular closed maps, *gpr*-closed maps and *rg*-closed maps have been introduced and studied by Long [25], Gnanambal [30] and Arockiarani [23] respectively. In 2012, [31] we introduced the concepts of Strongly g^*-closed sets and Strongly g^*-open set in topological spaces. Also we have introduced the concepts of Strongly g^*-continuous functions, Strongly g^*-irresolute functions, Strongly g^*-open maps and Strongly g^*-closed maps in ([32], [33], [34], [35]).

In this paper, by deriving the properties of Strongly $T^{\#g}$-space, Strongly g^*C-homeomorphisms and Strongly g^*-connectedness and studied some of their properties. Further various characterisation are studied.

II. Preliminaries

Throughout this paper (X, τ) and (Y, σ) represent topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ), $cl(A)$, $int(A)$ and Ac denote the closure of A, the interior of A and the complement of A in X, respectively.

Definition 2.1. A subset A of a topological space (X, τ) is called

(a) a preopen set [11] if $A \subseteq int(cl(A))$ and pre-closed set if $cl(int(A)) \subseteq A$.

(b) a semiopen set [10] if $A \subseteq cl(int(A))$ and semi closed set if $int(cl(A)) \subseteq A$.

(c) an α-open set [12] if $A \subseteq int(cl(int(A)))$ and an α-closed set if $cl(int(cl(A))) \subseteq A$.

(d) a semi-preopen set [36] (β-open set) if $A \subseteq cl(int(cl(A)))$ and semi-preclosed set if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$.

Definition 2.2. A space (X, τ_X) is called a $T_{½}$-space [7] if every g-closed set is closed.

Definition 2.3. [31] Let (X, τ) be a topological space and A be its subset, then A is Strongly g^*-closed set if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and U is g-open.

The complement of Strongly g^*-closed set is called Strongly g^*-open set in (X, τ).

Definition 2.4. [32] Let X and Y be topological spaces. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be strongly $G*$-continuous (sg^*-continuous) if the inverse image of every open set Y is sg^*-open in X.

Definition 2.5. [33] Let X and Y be topological spaces. A map $f : (X, \tau) \rightarrow (\text{Y}, \sigma)$ is said to be strongly g^*-irresolute map (sg^*-irresolute map) if the inverse image of every sg^*-open set in Y is sg^*-open in X.

Definition 2.6. [33] Let X and Y be two topological spaces. A bijection map f : $(\text{X}, \tau) \rightarrow (Y, \sigma)$ from a topological space X into a topological space Y is called strongly g^*-Homeomorphism (sg^*-homeomorphism) if f and f^{-1} are sg^*-continuous.

Definition 2.7. [34] (a) Let X be a topological space and let $x \in X$. A subset N of X is said to be Strongly g^*-nbbd of x if there exists an Strongly g^*-open set G such that $x \in G \subset N$.

The collection of all Strongly g^*-nbhd of $x \in X$ is called a Strongly g^*-nbhd system at x and shall be denoted by Strongly g^* $N(x)$.

(b) Let X be a topological space and A be a subset of X, A subset N of X is said to be Strongly g^*-*nbhd* of A if there exists a Strongly g^*-open set G such that $A \in G \subseteq N$.

(c) Let A be a subset of X. A point $x \in A$ is said to be a Strongly g^*-interior point of A, if A is a Strongly g^* $N(x)$. The set of all Strongly g^*-interior points of A is called a Strongly g^*-interior of A and is denoted by Sg^* $INT(A)$.

Sg^* INT $(A) = \cup\{G : G$ is *Strongly* g^*-open, $G \subset A\}$.

(d) Let A be a subset of X. A point $x \in A$ is said to be a Strongly g^*-closure of A. Then

$Sg^*C\ L(A) = \bigcap\{F : A \subset F \in$ Strongly $g^*C\ (X, \tau)\}$.

Definition 2.8. [35] A topological space (X, τ) is said to be

(a) Strongly-$T_0^{g^*}$ if for each pair of distinct points x, y in X, there exists a Strongly

g^*-open set U such that either $x \in U$ and $y \notin U$ or $x \notin U$ and $y \in U$.

(b) Strongly-$T_1^{g^*}$ if for each pair of distinct points x, y in X, there exist two Strongly g^*-open sets U and V such that $x \in U$ but $y \notin U$ and $y \in V$ but $x \notin V$.

(c) Strongly-$T_2^{g^*}$ if for each distinct points x, y in X, there exist two disjoint Strongly g^*-open sets U and V containing x and y respectively.

Definition 2.9. [35] A topological space (X, τ) is said to be Strongly g^*-symmetric if for x and y in X, $x \in Sg^*\ C\ L(\{y\})$ implies $y \in Sg^*\ C\ L(\{x\})$.

Definition 2.10. [35] A function $f : (X, \tau) \to (Y, \sigma)$ is called a Strongly g^*-open function if the image of every Strongly g^*-open set in (X, τ) is a Strongly g^*-open set in (Y, σ).

Definition 2.11. [37] A subset A of a topological space (X, τ) is called a Strongly g^*-Difference set (briefly, Strongly $g^*d_\#$-set) if there are $U, V \in$ Strongly $g^*O(X, \tau)$ such that $U \neq X$ and $A = U/V$.

It is true that every Strongly g^*-open set U different from X is a Strongly $g^*d_\#$-set if $A = U$ and $V = \varphi$. So, we can observe the following.

Definition 2.12. [37] A topological space (X, τ) is said to be

(a) Strongly $\tilde{}d^0$-G^* if for any pair of distinct points x and y of X there exists a Strongly $g^*d_\#$-set of X containing x but not y or Strongly $g^*d_\#$-set of X containing y but not x.

(b) Strongly $\tilde{}d^1$-$G*$ if for any pair of distinct points x and y of X there exists a Strongly $g^{*d}{}_\#$-set of X containing x but not y and Strongly $g^*d_\#$-set of X containing y but not x.

(c) Strongly $\tilde{}d_2$-$G*$ if for any pair of distinct points x and y of X there exist dis-joint Strongly $g^*\ d_\#$-set G and E of X containing x and y, respectively.

Definition 2.13. [37] A point $x \in X$ which has only X as the Strongly g^*-neighbourhood is called a Strongly g^*-neat point.

Definition 2.14. [37] (a) The intersection of all Strongly g^*-open subsets of (X, τ) containing A is called the Strongly g^*-kernel of A (briefly, S-g^*-$K_\#(A)$).

S-g^*-$K_\#(A) = \bigcap\{G \in$ Strongly $g^*\ (X, \tau) : A \subseteq G\}$.

(b) Let $x \in X$. Then Strongly g^*-kernel of x is denoted by S-g^*-$K_\#(\{x\}) = \bigcap\{G \in$

Strongly $g^*\ (X, \tau) : x \in G\}$.

III. Strongly $T_\#$G-Space via Strongly G^*-Open Sets*

Definition 3.1. A space (X, τ) is called Strongly $T_\#$g-space if every Strongly g^*-closed set is closed.

Theorem 3.1. For a topological space (X, τ) the following conditions are equivalent.

(a) (X, τ) is a Strongly $T_\#$g-space.

(b) Every singleton $\{x\}$ is either g-closed (or) clopen.

Proof. $(a) \Rightarrow (b)$ Let $x \in X$. Suppose {x} is not a Strongly g^*-closed set of (X, τ). Then $X - \{x\}$ is not a g-open set. Thus $X - \{x\}$ is a g-closed set of (X, τ). Since (X, τ) is a Strongly $T^{\#g^*}$-space, $X - \{x\}$ is a g-closed set of (X, τ), i.e., {x} is g-open set of (X, τ).

$(b) \Rightarrow (a)$ Let A be a g-closed set of (X, τ). Let $x \in int(cl(A))$ by (b), $\{x\}$ is either g-closed (or) clopen.

Case(i): Let $\{x\}$ be a g-closed. If we assume that $x \notin A$, then we would have $x \in int(cl(A)) - A$ which cannot happen. Hence $x \in A$.

Case(ii): Let $\{x\}$ be a clopen. Since $x \in int(cl(A))$, then $\{x\} \cap A = \varphi$. This shows that $x \in A$. So in both cases we have $int(cl(A)) \subseteq A$. Trivially $A \subseteq int(cl(A))$. Therefore $A = int(cl(A))$ (or) equivalently A is clopen. Hence (X, τ) is a Strongly T #g-space.

Proposition 3.3. If $f : (X, \tau) \rightarrow (Y, \sigma)$ be a Strongly g^*-continuous function and (X, τ) be a Strongly $T^{\#g^*}$-space. Then f is continuous.

Proof. Let V be closed set in (Y, σ). Since f is a Strongly g^*-continuous function, $f^{-1}(V)$ is a Strongly g^*-closed set in (X, τ). Since (X, τ) is a Strongly $T^{\#g^*}$-space, $f^{-1}(V)$ is closed set in (X, τ). Hence f is continuous.

Theorem 3.2. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a mapping and (X, τ) be a Strongly $T^{\#g^*}$-space, then f is continuous if one of the following conditions is satisfied.

(a) f is Strongly g^*-continuous.

(b) f is Strongly g^*-irresolute.

Proof. Obvious.

Theorem 3.3. A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is a Strongly g^*-continuous function if and only if the inverse image of every open set in (Y, σ) is a Strongly g^*-open set in (X, τ).

Proof. Necessity : Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a Strongly g^*-continuous function and U be an open set in (Y, σ). Then $Y - U$ is closed in (Y, σ). Since f is a Strongly g^*-continuous function, $f^{-1}(Y - U) = X - f^{-1}(U)$ is a Strongly g^*-closed in (X, τ) and hence $f^{-1}(U)$ is a Strongly g^*-open in (X, τ).

Sufficiency: Assume that $f^{-1}(V)$ is a Strongly g^*-open set in (X, τ) for each open set V in (Y, σ). Let V be a closed set in (Y, σ). Then $Y - V$ is an open set in (Y, σ). By assumption, $f^{-1}(Y - V) = X - f^{-1}(V)$ is a Strongly g^*-open in (X, τ), which implies that f^{-1} (V) is a Strongly g^*-closed set in (X, τ). Hence f is a Strongly g^*-continuous function.

Proposition 3.6. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be any topological space and (Y, σ) be a Strongly T #g-space. If $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ are Strongly g^*-continuous functions, then their composition $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is a Strongly g^*-continuous function.

Proof. Let V be a closed set in (Z, η). Since $g : (Y, \sigma) \rightarrow (Z, \eta)$ is a Strongly g^*-continuous function, $g^{-1}(V)$ is a Strongly g^*-closed set in (Y, σ). Since (Y, σ) is a∗ Strongly $T^{\#g^*}$-space, $g^{-1}(V)$ is a closed set in (Y, σ). Since $f : (X, \tau) \rightarrow (Y, \sigma)$ is a Strongly g^*-continuous function, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is a Strongly g^*-closed set in (X, τ). Hence $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is a Strongly g^*-continuous function.

Definition 3.2. A function $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ is said to be a Strongly g^* C-homeomorphism if both f and f^{-1} are Strongly g^*-irresolute.

We denote the family of all Strongly g^* C-homeomorphisms of a topological space

(X, τX) onto itself by Strongly g^* C-$h(X, \tau_1)$.

Proposition 3.8. Let $f : (X, \tau_1) \to (Y, \tau_2)$ and g : $(Y, \tau_2) \to (Z, \tau_3)$ are Strongly g^*C-homeomorphisms, then their composition $g \circ f : (X, \tau_1) \to (Z, \tau_3)$ is also Strongly g^* C *-homeomorphism*.

Proof. Let U be a Strongly g^*-open set in $(Z, \tau3)$. Since g is Strongly g^*-irresolute, $g^{-1}(U)$ is Strongly g^*-open in (Y, τ_2). Since f is Strongly g^*-irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is Strongly g^*-open set in (X, τ_1). Therefore g ∘ f is Strongly g^*-irresolute.

Also for a Strongly g^*-open set G in (X, τ_1), we have $(g \circ f)(G) = g(f(G)) = g(W)$, where $W = f(G)$. By hypothesis, $f(G)$ is Strongly g^*-open in (Y, τ_2) and so again by hypothesis, $g(f(G))$ is a Strongly g^*-open set in (Z, τ_3). That is $(g \circ f)(G)$ is a Strongly g^*-open set in (Z, τ_3) and therefore $(g \circ f)^{-1}$ is Strongly g^*-irresolute. Also $g \circ f$ is a bijection. Hence g ∘ f is Strongly g^*C-homeomorphism.

Theorem 3.4. The set Strongly g^* C-$h(X, \tau_1)$ is a group under the composition of maps.

Proof. Define a binary operation $*$: Strongly g^*C-$h(X, \tau_1) \times$Strongly g^*C-$h(X, \tau_1) \to$ Strongly g^*C-$h(X, \tau_1)$ by $f * g = g \circ f$ for all $f, g \in$ Strongly g^* C-$h(X, \tau_1)$ and ∘ is the usual operation of composition of maps. g ∘ $f \in$ Strongly g^* C-$h(X, \tau_1)$.

We know that the composition of maps is associative and the identity map $I : (X, \tau_1) \to (X, \tau_1)$ belonging to Strongly g^*C-$h(X, \tau_1)$ serves as the identity element. If $f \in$ Strongly g^* C-$h(X, \tau_1)$, then $f^{-1} \in$ Strongly g^* C-h(X, τ_1) such that $f \circ f^{-1} = f^{-1} \circ f = I$ and so inverse exists for each element of Strongly g^* C-$h(X, \tau_1)$. Therefore (Strongly g^* C-$h(X, \tau_1)$, ∘) is a group under the operation of composition of maps.

Theorem 3.5. Let f : $(X, \tau_1) \to (Y, \tau_2)$ be a Strongly g^* C-homeomorphism. Then f induces an isomorphism from the group Strongly g^* C-$h(X, \tau_1)$ onto the group Strongly g^* C-$h(Y, \tau_2)$.

Proof. Using the map f, we define a map Ψ_f : Strongly g^* C-h(X, τ_1) → Strongly g^* C - $h(Y, \tau_2)$ by $\Psi_f(h) = f \circ h \circ f^{-1}$ for every h ∈ Strongly g^* C-$h(X, \tau_1)$. Then Ψf is a bijection. Further, for all $h1$, $h2 \in$ Strongly g^* C-$h(X, \tau_1)$, $\Psi_f(h1 \circ h2) = f \circ (h1 \circ h2) \circ f^{-1} = (f \circ h1 \circ f^{-1}) \circ (f \circ h2 \circ f^{-1}) = \Psi_f(h1) \circ \Psi_f(h2)$.

Therefore, Ψ_f is a homeomorphism and so it is an isomorphism induced by f.

Definition 3.3. A topological space (X, τ_1) is said to be Strongly g^*-connected if X cannot be expressed as a disjoint union of two non-empty Strongly g^*-open sets. A subset of X is Strongly g^*-connected if it is Strongly g^*-connected as a subspace.

Theorem 3.6. For a topological space (X, τ_1), the following are equivalent.

(a) (X, τ_1) is Strongly g^*-connected.

(b) (X, τ_1) and φ are the only subsets of (X, τ_1) which are both Strongly g^*-open and Strongly g^*-closed.

(c) Each Strongly g^*-continuous map of (X, τ_1) into a discrete space (Y, τ_2) with at least two points is constant map.

Proof. (a) ⇒ (b): Suppose (X, τ_1) is Strongly g^*-connected. Let S be a proper sub-set which is both Strongly g^*-open and Strongly g^*-closed in (X, τ_1). Its complement X/S is also Strongly g^*-open and Strongly g^*-closed. $X = S \cup$ (X/S), a disjoint union of two non empty Strongly g^*-open sets which is contradicts (a). Therefore $S = \varphi$ or X .

$(b) \Rightarrow (a)$: Suppose that $X = A \cup B$ where A and B are disjoint non empty Strongly g^*-open subsets of (X, τ_1). Then A is both Strongly g^*-open and Strongly g^*-closed. By assumption $A = \varphi$ or X . Therefore X is Strongly g^*-connected.

(b) ⇒ (c): Let $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ be a Strongly g^*-continuous map. Then (X, τ_1) is covered by Strongly g^*-open and Strongly g^*-closed covering $\{ f^{-1}(y) : y \in Y \}$. By assumption $f^{-1}(y) = \varphi$ or X for each $y \in Y$. If $f^{-1}(y) = \varphi$ for all $y \in Y$, then f fails to be a map. Then there exists only one point $y \in Y$ such that $f^{-1}(y) = \varphi$ and hence $f^{-1}(y) = X$. This shows that f is a constant map.

$(c) \Rightarrow (b)$: Let S be both Strongly g^*-open and Strongly g^*-closed in X. Suppose $S = \varphi$. Let f : $(X, \tau_1) \rightarrow (Y, \tau_2)$ be a Strongly g^*-continuous map defined by $f(S) = y$ and $f(S^c) = \{\omega\}$ for some distinct points y and ω in (Y, τ_2). By assumption f is a constant map. Therefore we have $S = X$.

Theorem 3.7. Every Strongly g^*-connected space is connected.

Proof. Let (X, τ_1) be Strongly g^*-connected. Suppose X is not connected. Then there exists a proper non empty subset B of (X, τ_1) which is both open and closed in (X, τ_1). Since every closed set is Strongly g^*-closed, B is a proper non empty subset of (X, τ_1) which is both Strongly g^*-open and Strongly g^*-closed in (X, τ_1), (X, τ_1) is not Strongly g^*-connected. This proves the theorem.

Theorem 3.8. If $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ is a Strongly g^*-continuous and X is Strongly g^*-connected, then (Y, τ_2) is connected.

Proof. Suppose that (Y, τ_2) is not connected. Let $Y = A \cup B$ where A and B are disjoint non-empty open set in (Y, τ_2). Since f is Strongly g^*-continuous and onto, $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty Strongly g^*-open sets in (X, τ_1).

This contradicts the fact that (X, τ_1) is Strongly g^*-connected. Hence Y is connected.

Theorem 3.8. If $f : (X, \tau_1) \rightarrow (Y, \tau_2)$ is a Strongly g^*-irresolute and (X, τ_1) is Strongly g^*-connected, then (Y, τ_2) is Strongly g^*-connected.

Proof. Suppose that (Y, τ_2) is not Strongly g^*-connected. Let $Y = A \cup B$ where A and B are disjoint non-empty Strongly g^*-open sets in (Y, τ_2). Since f is Strongly g^*-irresolute and onto, $X = f^{-1}(A) \cup f^{-1}(B)$ where $f^{-1}(A)$ and $f^{-1}(B)$ are disjoint non-empty Strongly g^*-open sets in (X, τ_1).

This contradicts the fact that (X, τ_1) is Strongly g^*-connected. Hence (Y, τ_2) is Strongly g^*-connected.

Theorem 3.10. Suppose that (X, τ_1) is Strongly T #g if and only if it is Strongly g^* -connected.-space then (X, τ_1) is connected

Proof. Suppose that (X, τ_1) is connected. Then (X, τ_1) cannot be expressed as disjoint union of two non-empty proper subsets of (X, τ_1). Suppose (X, τ_1) is not a Strongly g^*-connected space. Let A and B be any two Strongly g^*-open subsets of (X, τ_1) such that $Y = A \cup B$, where $A \cap B = \varphi$ and $A \subset X$, $B \subset X$ Since (X, τ_1) is Strongly∗T #g-space and A, B are Strongly g^*-open. A, B are open subsets of (X, τ_1), which contradicts that (X, τ_1) is connected. Therefore (X, τ_1) is Strongly g^*-connected.

Conversely, every open set is Strongly g^*-open. Therefore every Strongly g^*-connected space is connected.

Theorem 3.11. If the Strongly g^*-open sets C and D form a separation of (X, τ_1) and if (Y, τ_2) is Strongly g^*-connected subspace of

(X, τ_1), then (Y, τ_2) lies entirely within C or D.

Proof. Since C and D are both Strongly g^*-open in (X, τ_1), the sets $C \cap Y$ and $D \cap Y$ are Strongly g^*-open in (Y, τ_2). These two sets are disjoint and their union is (Y, τ_2). If they were both non-empty, they would constitute a separation of (Y, τ_2).

Therefore, one of them is empty. Hence (Y, τ_2) must lie entirely in C or in D.

Theorem 3.11. Let A be a Strongly g^*-connected subspace of (X, τ_1). If $A \subset B \subset Sg^*\ C\ L(A)$ then B is also Strongly g^*-connected.

Proof. Let A be Strongly g^*-connected and let $A \subset B \subset Sg^*\ C\ L(A)$. Suppose that $B = C \cup D$ is a separation of B by Strongly g^*-open sets. A must lie entirely in C or in D. Suppose that $A \subset C$, then $Sg^*\ C\ L(A) \subset Sg^*\ C\ L(C)$. Since $Sg^*\ C\ L(C)$ and D are disjoint, B cannot intersect D.

This contradicts the fact that D is non empty subset of B. So $D = \varphi$ which implies B is Strongly g^*-connected.

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