



## **Comparisons of Heuristic, Case-Based, and Polynomial Approach to Solve n-Queens Problem**

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### **ABSTRACT**

*The N-queen problems is an intractable problem and for a large value of 'n' the problem cannot be solved in polynomial time and thus is placed in 'NP' class. Various computational approaches are present to solve the N-Queen problem and no efficient algorithm for this type of problem has yet been found. In this paper we have tried to compare case based approach, polynomial approach and heuristic approach to solve the N-queen problem.*

### **I. INTRODUCTION**

The problems in computer science can be grouped in different classes on the basis of time required to find a correct solution. The class 'P' contains all those decision problems for which polynomial time algorithms exist. The problems which can be solved in polynomial time on a non-deterministic computer are placed in 'NP' class. However, solution to a problem in 'NP' can be verified in polynomial time<sup>[1]</sup>.

The central problem in the engineering field is the optimization problem. If the variables in the optimization problem are discrete rather than continuous, we call it combinatorial optimization problem. In the combinatorial optimization problem, the number of elements in the configuration space is factorially large, therefore we cannot explore them exhaustively.

The N-Queen problem is a well known combinatorial optimization problem which is an NP Complete and there are several approaches to solve it. None of the approach is completely efficient. For the purpose of finding the answers, n-queen problem is classified in 3 classes: 1. Finding all answers 2. Finding some answers and 3. Finding the first answer<sup>[2]</sup>.

### **II. N-QUEEN PROBLEM**

The N-Queen problem can be described as following : try to find a solution to place N queens on a chessboard so that no queen would attack any other queen, which means that none of them share a common row, column, or is diagonal to one another. The 8-Queen problem

is a traditional example of N-Queen problem, which was originally proposed in 1848 by the chess player Max Bezzel, and over the years, many mathematicians, including Gauss have worked on this puzzle and its generalised N-Queen problem<sup>[3]</sup>. However, in view of the complexity of the problem, hand calculations were not component for it. When turning to computer, it can easily find solutions using a recursive algorithm. But the costing time will increase in geometric index with the increase of problem scale. As of today, the N-queen problem is solved for N=26. There are exactly 22,317,699,616,364,044 ways to place 26 non-attacking queen on a 26x26 chessboard. Table I[4] shows number of solutions and unique solutions possible for placing N-Queens for N=[21, 26] for estimation of complexity.

**Table I No. of Solutions for Queens=[21,26]**

Problem Size N	Overall Solutions	Fundamental Solutions
21	314666222712	39333324973
22	2691008701644	336376244042
23	24233937684440	3029242658210
24	227514171973736	28439272956934
25	2207893435808352	275986683743434
26	22317699616364044	2789712466510289

### III. ALGORITHM FOR EACH APPROACHES

#### A. Genetic Algorithm

Genetic algorithm originated from evolutionary theory of Darwin and Mendel's genetic theory<sup>[5]</sup>. It is a random searching method. Since Professor John Holland first established the genetic algorithm in 1975. Genetic algorithms are search and optimization heuristic techniques based on the natural evolution process. The space solution is represented as a population, which consists of individuals that are evaluated using a fitness

function representing the problem being optimized. The basic structure of a genetic algorithm is shown in Figure 1. In each iteration (generation) of algorithm, a certain number of best-ranking individuals (chromosomes) is selected in the manner to create new better individuals (children). The children are created by some type of recombination (crossover) and they replace the worst-ranked part of the population. After the children are obtained, a mutation operator is allowed to occur and the next generation of the population is created. The process is iterated until the evolution condition terminates.

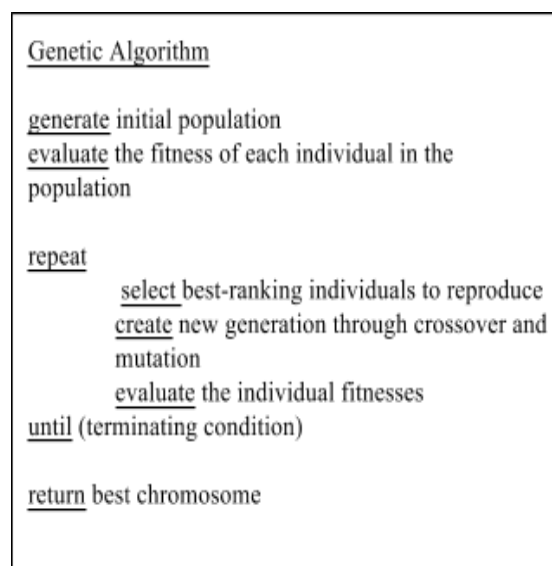


Figure 1. Structure of Genetic Algorithm

#### B. Rakhya's Method

Rakhya's method is recently introduced where the solutions to the given problem are found with the help of two operations named as jiggling and shifting<sup>[6]</sup>. These operations are performed on the predefined arrangement of queens, known as A & B patterns. The solutions are obtained in two phases. In the first phase, a definite solution is found using a definite operation set. In the second phase, other solution is generated from the solution achieved in the prior phase. It provides the solution for all values of n>3. There are six

best option when it comes about finding all possible solutions. This approach suffers from exponential growth of time and steps as n increased. Figure 2 plots the number of steps needed to compute the first solution of N by backtracking method. In order to represent the variation we have taken the log value of number of steps .

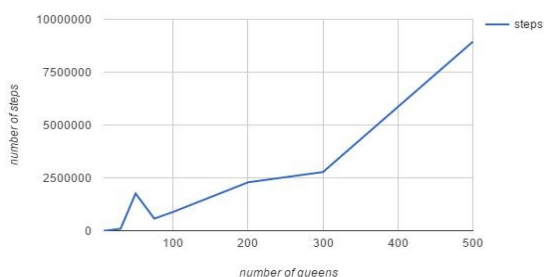


Figure 3: Fitness function graph of Genetic Algorithm

Case based approaches like Rakhya's method suffers a major drawback that the proposed solutions has its limitations to the certain class of solutions. In this method solutions to the given problem are found with the help of two operations named jiggling and shifting. These operations are performed on the predefined arrangement of queens, known as Rakhya (A & B) patterns. These predefined arrangement limits the potential of algorithm to find all solutions. These drawbacks with polynomial and case based approaches was the first motivation to try evolutionary algorithm. a genetic population-based metaheuristic optimization algorithm inspired by biological evolution where calculation of fitness function takes the most time Figure 3 plots the average number of fitness function computation in 10 runs

In order to compare the three approaches we have plotted the log of number of steps to determine the first solution. Since number of steps may vary in the each execution of the genetic approach we have considered the average of the first 10 runs. Figure 4 comparisons of three approaches.

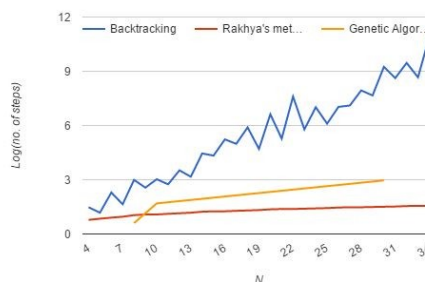


Figure 4. Comparisons of three approaches

Clearly from the plot in Figure 4 backtracking method has the highest resource consumption but it can evaluate all possible solutions for some values of n. Rakhya's method has lowest resource consumption but number of solution is limited. Number of steps in evolutionary algorithm varies but they lie in between other two approaches.

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- [3] Jordan Bell, Brett Stevens, "A survey of known results and research areas for n-queens", Discrete Mathematics 309 (2009), Available online at sciencedirect.com
- [4] Queens @ TUD <http://queens.inf.tu-dresden.de/>
- [5] Zhou Ming, Sun Shudong, Genetic

cases in that operation set based on the value of n. These cases are actually six set of numbers. The union of these sets is a set of all integers greater than 3.

The principle behind this approach is the principle of consecutive correction i.e., if there is any error in the present arrangement of queens, a correction is done and the new arrangement is checked again for errors. The arrangement in which no more correction is required is said to be the final solution. Rakhya's method not only provides the solution in minimum possible steps but it also enables us to achieve other final solutions from the already obtained solutions. Rakhya (A) and Rakhya (B) patterns these two patterns achieve conflict free positions for all the queens. They are designed to remove the vertical and horizontal conflicts, although there might still be one or more diagonal conflicts left. Jiggling and shifting operations are used in the second level, to remove the diagonal conflicts. Applications of these conflicts does not hinder the vertical and horizontal conflict free arrangement of queens attained in the first level. Jiggling is always performed in pairs. Tabular representation of equations of different cases and their respective operations set is given in Table 2. where N = no. of queens.

**Table II : Cases and Their Respective Operations**

CASE	EQUATIONS	OUTPUT	METHODS
A	$N \neq 6K+2$	EVEN	R(A)
B	$N \neq 6K+3$	ODD	R(B)
C	$N=6K+2$ , $N/2=0$	EVEN	R(A) J(r) or R(A) J(l)
D	$N=6K+2$ , $N/2 \neq 0$	ODD	R(A) J(r) S(r) OR R(A) J(l) S(l)
E	$N=6K+3$ , $((N+1)/2) \neq 0$	ODD	R(B) J(r) S(R) S(R)
F	$N=6K+3$ , $((N+1)/2)=0$	EVEN	R(B) J(l) S(L) S(L)

### C. Backtracking Method

The term "backtrack" was coined by American mathematician D. H. Lehmer in 1950s. Backtracking is a general algorithm for finding all(or some) solutions to some computational problems, notably constraint satisfaction problems, that incrementally builds candidates to the solutions, and abandons each partial candidate c(backtrack), as soon as it determines that c cannot possibly be completed to a valid solution. Backtracking depends on user-given "black box procedures" that defines the problem to be solved, the nature of the partial candidates, and how they are extended into complete candidates. It is therefore a metaheuristic rather than a specific algorithm<sup>[7]</sup>.

Method of backtracking is frequently used to get solutions of n-queen problem as it provides solution for all n. We can place a queen on the top left corner of nxn matrix then we find a conflict free location in the next column and place the second queen there. This process continues for the remaining queens and thus a solution is achieved.

### III. COMPARISON AND ANALYSIS

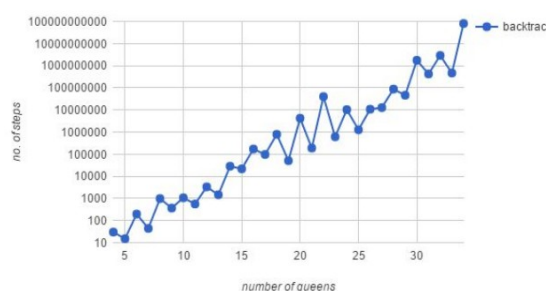


Figure 2. Number of steps to find first solution using backtracking

Different approaches yields different outcomes i.e., finding all answers, finding some answers, and finding the first answers. Backtracking method can provide all possible solutions that any n can have. This will be the

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