Volume 5, Issue 1, January 2018 ISSN: 2348-8565 (Online)
International Journal of Modern Engineering and Research Technology
Website: http://www.ijmert.org Email: editor.ijmert@gmail.com

On Fuzzy Supra b-open map and Fuzzy Supra b-closed map

Mukta Bhatele
Professor
Department of Computer Science & Engineering
Gyan Ganga Institute of Technology & Sciences
Jabalpur (M.P.), [INDIA]
Email: mukta_bhatele@rediffmail.com

Madhulika Shukla
Associate Professor
Gayan Ganga Insitude of Technology and Sciences
Jabalpur (M.P.), [INDIA]
Email: shukla.madhulika07@gmail.com

ABSTRACT

In this paper, we introduce and investigate a new class of sets and maps between fuzzy topological spaces called fuzzy supra b-open sets, fuzzy supra b-open maps and fuzzy supra b-closed maps respectively. Furthermore, we investigate several properties of fuzzy supra b -open maps and fuzzy supra b-closed maps.

2000 Mathematics Subject Classifications: 54A10, 54A20

Keywords:—*fuzzy Supra b-open set, fuzzy b-open map, fuzzy Supra b-closed map and fuzzy Supra topological space.*

I. INTRODUCTION

The notion of fuzzy sets was introduced by L. A. Zadeh [11] in 1965. C. L. Chang [4] has extended the concept of topology by taking a collection of fuzzy sets instead of crisp sets and developed the theory of Fuzzy Topological spaces. In 1983, A. S. Mashhour et al. [6] introduced the concept of supra topological spaces and studied s-continuous functions and s∗- continuous functions. In 2008, Devi et al. [5] introduced supra α-open (closed) sets. In 1987, M. E. Abd El-Monsef et al. [2] introduced fuzzy supra topological spaces and studied fuzzy supracontinuous functions.

In this paper we introduce and study the new class of mappings called fuzzy supra b-open maps and fuzzy supra b-closed maps in fuzzy topological spaces. Also we define the relation between them.

2. PRELIMINARIES

Let X be a non empty set. A collection τ of fuzzy sets in X is called a fuzzy topology on X if the whole fuzzy set 1 and the empty fuzzy set 0 is the members of τ and τ is closed with respect to any union and finite intersection. The members of τ are called fuzzy open sets and the complement of a fuzzy open set is called fuzzy closed set. The closure of a fuzzy set λ (denoted by (λ) is the intersection of all fuzzy closed which contains λ. The interior of a fuzzy setλ (denoted by int(λ))is the union of all fuzzy open subsets of λ. A fuzzy set λ in X is fuzzy open (resp. fuzzy closed) if and only int(λ) = λ (resp. cl(λ) = λ).

Definition 2.1: Let (X, τ) be a fuzzy topological space. A fuzzy set λ in the space X is called:

(i) Fuzzy semi-open set [1] if λ ≤ cl (int(λ)) and semi-closed fuzzy set if int(cl(λ)) ≤ λ.

(ii) Fuzzy pre-open set [3] if λ ≤ int(cl (λ)) and pre-closed fuzzy set if cl (int(λ)) ≤ λ.

(iii) Fuzzy α-open set [3] if λ ≤ int(cl (int(λ))) and α-closed fuzzy set if cl(int(cl(λ))) ≤ λ.

(iv) Fuzzy supra semi-open set [2] if $\lambda \leq cl^{\mu}(int^{\mu}(\lambda))$ and fuzzy supra semi-closed set if $int^{\mu}(cl^{\mu}(\lambda)) \leq \lambda$.

(v) Fuzzy supra α-open set [2] if $\lambda \leq int^{\mu}(cl^{\mu}(int^{\mu}(\lambda)))$ and fuzzy supra α-closed set if $cl^{\mu}(int^{\mu}(cl^{\mu}(\lambda))) \leq \lambda$.

(vi) Fuzzy Supra-open→ Fuzzy Supra α-open→Fuzzy supra semi-open→ Fuzzy Supra b-open

3. FUZZY SUPRA B-OPEN SETS

Definition 3.1: Let (X, μ) be a fuzzy supra topological space. A set λ is called a fuzzy supra b-open (fuzzy supra b-closed) set in $\lambda \leq cl^{\mu}(int^{\mu}(\lambda)) \cup int^{\mu}(cl^{\mu}(\lambda))$(resp.$cl^{\mu}(int^{\mu}(\lambda)) \cap int^{\mu}(cl^{\mu}(\lambda)) \leq \lambda$).

Theorem 3.2: Every fuzzy supra semi-open set is fuzzy supra b-open.

Proof: Let λ be a fuzzy supra semi-open set in (X, μ). Then $\lambda \leq cl^{s}(int^{s}(\lambda))$. Hence $\lambda \leq cl^{s}(int^{s}(\lambda)) \cup int^{s}(cl^{s}(\lambda))$ and λ is fuzzy supra b-closed set in (X, μ).

The converse of the above theorem need not be true as shown by following example.

Example 3.3: Let (X, μ) be a fuzzy supra topological space, where $X = \{x_1, x_2\}$. Let λ, ϑ be fuzzy sets in X and $\mu = \{0, \lambda, \vartheta, 1\}$ defined as $\lambda(x_1) = 0.2$, $\lambda(x_2) = 0.3$, $\vartheta(x_1) = 0.5$ and $\vartheta(x_2) = 0.6$. Here $\gamma(x_1) = 0.4$ and $\gamma(x_2) = 0.5$ is a fuzzy supra b-open set, but it is not fuzzy supra semi-open.

DIAGRAM

Fuzzy Supra-open→Fuzzy Supra α-open→Fuzzy supra semi-open→ Fuzzy Supra b-open

Theorem 3.4:

Arbitrary Union of fuzzy supra b-open sets is always fuzzy supra b-open.

Finite intersection of fuzzy supra b-closed sets may fail to be fuzzy supra b-open.

Xis a fuzzy supra b-open set.

Proof: (i) Let λ and ϑ be two fuzzy b-open sets. Then, $\lambda \leq cl^{\mu}(int^{\mu}(\lambda)) \cup int^{\mu}(cl^{\mu}(\lambda))$ and $\vartheta \leq cl^{\mu}(int^{\mu}(\lambda)) \cup int^{\mu}(cl^{\mu}(\vartheta))$. Then $\lambda \cup \vartheta \leq cl^{\mu}(int^{\mu}(\lambda \cup \vartheta)) \cup int^{\mu}(cl^{\mu}(\lambda \cup \vartheta))$. Therefore, $\lambda \cup \vartheta$ is fuzzy supra b-open sets.

(ii) As the definition.

Theorem 3.5: (i) Arbitrary intersection of fuzzy suprab-closed sets is always fuzzy supra b-closed.

(ii) Finite union of fuzzy supra b-closed sets may fail to be fuzzy supra b-closed.

Proof: As above theorem.

Definition 3.6: The fuzzy supra b-closure of a set λ, denoted by $cl_b^{\mu}(\lambda)$is the intersection of fuzzy suprab-closed sets including λ. The supra b-interior of a set λ, denotes by $int\_b^{\mu}(\lambda)$, is the union of fuzzy supra b-open sets included in λ.

Remark 3.7: It is clear that $int\_b^{\mu}(\lambda)$ is a fuzzy supra b-open set and $cl\_b^{\mu}(\lambda)$ is a fuzzy supra b-closed set.

$\lambda \leq cl\_b^{\mu}(\lambda)$;and$\lambda = cl_b^{\mu}(\lambda)$ iff λ is a fuzzy supra b-closed set.

$int\_b^{\mu}(\lambda) \leq \lambda$; and$int_b^{\mu}(\lambda) = \lambda$ iff λ is a fuzzy supra b-open set.

$cl\_b^{\mu}(1-\lambda) = 1-int\_b^{\mu}(\lambda)$.

$int\_b^{\mu}(1-\lambda) = 1-cl\_b^{\mu}(\lambda)$.

Proof: Obvious.

Theorem 3.8: (i) $int\_b^{\mu}(\lambda) \cup int\_b^{\mu}(\vartheta) \leq int\_b^{\mu}(\lambda \cup \vartheta)$;

(ii) $cl\_b^{\mu}(\lambda \cap \vartheta) \leq cl\_b^{\mu}(\lambda) \cap cl\_b^{\mu}(\vartheta)$

Proof: obvious.

Proposition: The intersection of a fuzzy supra α-open set and a supra b-open set is a fuzzy supra b-open set.

Fuzzy Supra b-Open Maps and Fuzzy Supra b-Closed Maps

Definition 4.1: A map f:(X, τ)→(Y, σ) is called a fuzzy supra b-open (resp. fuzzy b-closed) if the image each open (resp. fuzzy closed) set in X is fuzzy supra b-open (resp. fuzzy supra b-closed) in (Y, ϑ).

Theorem 4.2: A map f:(X, τ)→(Y, σ) is fuzzy supra b-open if and only of f(int(λ)) ≤ int_b^{ϑ} (f (λ)) for each set λ in X.

Proof: Suppose that f is a fuzzy supra map. Since int(λ) ≤ λ, then f(int(λ)) ≤ f(λ). By hypothesis, f(int(λ)) is a fuzzy supra b-open set and int_b^s (f(λ)) is the largest fuzzy supra b-open set contained in f(λ). Hence f(int(λ)) ≤ int_b^s (f(λ)).

Conversely, suppose λ is an fuzzy open set in X. Then, f(int(λ)) ≤ int_b^s (f(λ)). Since int(λ) = λ, then f(λ) is a fuzzy supra b-open set in (Y, τ) and f is a fuzzy b-open map.

Theorem 4.3: A map f:(X, τ)→(Y, σ) is fuzzy supra b-closed if and only if cl_b^μ (f(λ)) ≤ f (λ), then cl_b^μ (f(λ)) ≤ f(cl(λ)) foe each set λ in X.

Proof: Suppose f is fuzzy supra b-closed map. Since for each set λ in X, cl(λ) is fuzzy closed set in X, then f(cl(λ)) is a supra b-closed set in Y. Also, since f(λ) ≤ f(cl(λ)), then cl_b^{μ} (f(λ)) ≤ f(cl(λ)).

Conversely, let λ be a fuzzy closed set in X.Sincecl_b^s (f(λ)) is the smallest fuzzy supra b -closed set containingf(λ), then f(λ) ≤ cl_b^{μ} (f (λ)) ≤ f(cl(λ)) = f(λ). Thus, f(λ) = cl_b^{μ} (f(λ)). Hence, f(λ) is a fuzzy supra b-closed set in Y. Therefore, f is a fuzzy supra b-closed map.

Theorem 4.4: Let (X, τ), (Y, σ) and (Z, ϑ) be three fuzzy topological space and f:(X, τ)→(Y, σ) and g:(Y, σ)→(Z, ϑ) be two maps. Then,

If gof is fuzzy supra b-open and f is fuzzy continuous surjective, then g is a fuzzy supra b -open map.

If gof is open and g is fuzzy supra b-continuous injective, then f is a fuzzy supra b-oprn map.

Proof:(i) Let λ be an open set in Y. Then, $f^{(-1)}$ (λ) is an fuzzy open set in X. Since gof is a fuzzy supra b-open map, then (gof)($f^{(-1)}$ (λ)) = g(f($f^{(-1)}$) (λ)) = g(λ) (because f is subjective) is a fuzzy supra b-open set in Z. Therefore, g is fuzzy supra b-open map.

Let λ bean fuzzy open setin X. Then, g(f(λ)) is an fuzzy open set in Z. Therefore, g^(-1) (g(f (λ))) = f(λ) (because g is injective) is a fuzzy supra b-open set in Y. Hence f is a fuzzy supra b-open map.

Theorem 4.5: Let (X, τ) and (Y, σ) be two fuzzy topological spaces and f:(X, τ)→(Y, σ) be a bijective map. Then the following are equivalent:

fis a fuzzy supra b-open map.

fis a fuzzy supra b-closed map.

$f^{(-1)}$ is a fuzzy supra b-continuous map.

Proof: (i)⇒(ii) Suppose λ is a fuzzy closed set in X. Then 1-λ is an fuzzy open set in X and by (i), f(1-λ) is a fuzzy supra b-open set in Y. Since f is bijective, then f(1-λ)

= 1-f(λ). Hence, f(λ) is a fuzzy supra b-closed set in Y. Therefore, f is a fuzzy supra b-closed map.

(ii)⇒(iii) Let f is a fuzzy supra b-closed map and λ be fuzzy closed set in X. Since f is bijective then($f^{(-1)}$)($-^{1)}$(λ) = f(λ) which is a fuzzy supra b-closed set in Y. Theorem 7, f is fuzzy supra b-continuous map.

⇒(i) Let λ be an fuzzy open set in X. Since $f^{(-1)}$ is a fuzzy supra b-continuous map, then $(f^{\wedge(-1)})^{(-1)}(\lambda) = f(\lambda)$ is a fuzzy b-open set in Y. Hence, f is fuzzy supra b-open map.

REFERENCES:

[1] Azad K. K., On fuzzy semi-continuity, fuzzy almost continuity anf fuzzy weakly continuity, J. Math Apl. 82 (1981), 14-32.

[2] Abd El-Monsef M. E. and Ramadan A. E., "On fuzzy supra topological spaces", Indian J. Pure Appl. Math., 18 (4), pp. 322-329, (1987).

[3] Bin Shahna A. S., "On fuzzy strong semi continuity and fuzzy pre continuity", Fuzzy Sets and Systems, 44 pp.303- 308, (1991).

[4] Chang C.L., "Fuzzy topological spaces", Journal of Mathematical Analysis and Application, Vol. 24, pp.182- 190, (1968).

[5] Devi R., Sampath Kumar S. and Caldas M., "On Supra α- open sets and Sα-continuous functions" General Mathematics, Vol. 16, No. 2, pp.77-84, (2008).

[6] Mashhour A. S., Allam A. A., Mahmoud F. S and Khedr F. H., "On supra topological spaces", Indian J. Pure and Appl. Math. no.4, 14, pp. 502-510, (1983).

[7] Sahidul Ahmed and Biman Chandra Chetia, "On Certain Properties of Fuzzy Supra Semi open Sets", International Journal of Fuzzy Mathematics and Systems, Vol. 4, No 1, pp. 93-98, (2014).

[8] Sahidul Ahmed and Bimann Chandra Chetia, "Fuzzy Supra S-open And S-closed Mappings", International Journal of Fuzzy Mathematics and Systems, Vol. 5, No 1, pp. 41- 45, (2015).

[9] Sayed O. R., Takashi Noiri, "On Supra b-open sets and Supra b-continuity on topological spaces" European Journal of Pure and Applied Mathematics, Vol. 3, No.2, 295-302, (2010).

[10] ZadehL.A., Fuzzy Sets, Inform. and control, 8 (1965), 338-353.

* * * * *