



Application of Pattern-Output Viterbi Algorithm to Algebraic Soft-Decision Decoding Over Partial Response Channels

Dharkar Rupali Narhari

M.E. Research Scholar

*Department of Computer Science & Engineering,
M.B.E. Society's College of Engineering
Dr Babasaheb Ambedkar Marathwada University,
Ambajogai, (M.P.), [INDIA]
Email: anushreedashasahastra@yahoo.in*

Vaijanath V. Yerigeri

Head of the Department

*Electrical and Instrumentation Engineering
MBES's College of Engineering,
Dr Babasaheb Ambedkar Marathwada University,
Ambajogai, (M.S.) [INDIA]
Email: vaijanatha_y@rediffmail.com*

ABSTRACT

The algebraic soft-decision decoding algorithm (ASD) requires a reliability matrix as its input. In this paper, a new method to construct the reliability matrix over partial response (PR) channels of interest in magnetic recording is proposed by using recently introduced pattern-output Viterbi algorithm (POVA). A modified bit-level generalized minimum distance (BGMD) algorithm is also proposed with the POVA to achieve performance gains over PR channels that are as large as gains as over AWGN channels. The proposed work will carry out on re-configurable FPGA technology, by adopting parallel/pipeline features of the hardware resources. The overall system performance could be improved. The exiting algorithm is redesigned using HDL language, simulation, synthesis and implementation (translation, mapping place & routing) done with FPGA based EDA tools

Keywords:—*Reed-Solomon codes, soft-decision decoding, ASD algorithm, KV algorithm, pattern-output Viterbi algorithm, partial response channel, BGMD algorithm, BCJR algorithm.*

I. INTRODUCTION

In digital communication system, error detection and error correction is important for reliable communication. Error detection techniques are much simpler than forward error correction (FEC). But error detection techniques have certain disadvantages. Error detection pre supposes the existence of an automatic repeat request (ARQ) feature which provides for the retransmission of those blocks, segments or packets in which errors have been detected. This assumes some protocol for reserving time for the retransmission of such erroneous blocks and for reinserting the corrected version in proper sequence. It also assumes sufficient overall delay and corresponding buffering that will permit such reinsertion. The latter becomes particularly difficult in synchronous satellite communication where the transmission delay in each direction is already a quarter second. A further drawback of error detection with ARQ is its inefficiency at or near the system noise threshold. For, as the error rate approaches the packet length, the majority of blocks will contain detected errors and hence require retransmission, even several times, reducing the throughput drastically. In such cases, forward error correction, in addition

to error detection with ARQ, may considerably improve throughput. In order to enhance error correction capability, soft-decision decoding for Reed-Solomon (RS) codes has drawn significant research interest. The algebraic soft-decision decoding (ASD) algorithm, also known as the Koetter-Vardy (KV) algorithm [1], introduced how to use soft information based on the Guruswami-Sudan algorithm [2] for better error correction capability. The ASD algorithm requires, as its input, a reliability matrix which consists of symbol reliabilities. Over additive white Gaussian noise (AWGN) channels, reliabilities of bits in a symbol can be mapped into symbol reliability by multiplication of the bit error probabilities because bit errors are independent. However, over partial response (PR) channels of interest in magnetic recording systems, errors happen in patterns, so several erroneous bits may be strongly correlated. As a result, the symbol-based Bahl-Cocke-Jelinek-Raviv (BCJR) algorithm [3] is generally used to estimate symbol reliabilities in spite of its high complexity and long latency. Another limitation of the ASD algorithm is its small performance gain for high-rate codes (of interest in magnetic recording systems) even with infinite multiplicity. To reduce the complexity or/and to achieve better error correction capability, the ASD algorithm is combined with other soft-decision decoding algorithms [4] such as the Generalized Minimum Distance (GMD) algorithm [5].

In this paper, we propose a new reliability matrix construction method by using the probable error-pattern extracting Viterbi algorithm. A modified work will carry out on re-configurable FPGA technology, by adopting parallel/pipeline features of the hardware resources. The overall system performance could be improved such that it makes possible for the basic idea of the

BGMD to provide performance gain over PR channels similar to that over AWGN channels.

This implements the error correction method at real time while operating many-to-one and one-to-many data communication. The overall architecture primitive modules are designed, simulated, synthesized and structured in VHDL and by using various FPGA based EDA Tools

II. PATTERN-OUTPUT VITERBI ALGORITHM (POVA)

Viterbi Algorithm

The Viterbi decoding algorithm proposed in 1967 is a decoding process for Convolutional codes. Convolutional coding, as we all known, has been widely used in communication systems including deep space communications and wireless communications, such as IEEE 802.11a/g, WiMax, DAB/DVB, WCDMA and GSM.

Viterbi decoding is the best technique for decoding the convolutional codes but it is limited to smaller constraint lengths. The basic building blocks of Viterbi decoder are branch metric unit, add compare and select unit and survivor memory management unit. The two techniques for decoding the data are trace back (TB) method and Register Exchange (RE) method.

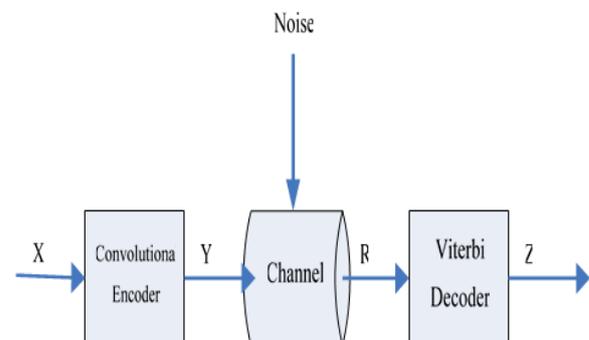


Figure 1: A Simple Viterbi Decoding System.

TB method is used for longer constraint lengths but it has larger decoding time. Also extra circuitry is required to reverse the decoded bits. The RE method is simpler and faster than the TB method for implementing the VD. RE method is not appropriate for decoders with long constraint lengths.

Recently, a modified Viterbi algorithm, the so called POVA, was proposed for pattern-based soft-decision decoding. The POVA provides the probable error pattern which terminates at every bit position and corresponding pattern reliability as well as hard-decision output over PR channels [6]. The probable error pattern output is determined by the difference over GF(2) between input sequences corresponding to two paths entering the survived state at every stage.

$$E_l = x_0 - x_1 \dots\dots\dots(1)$$

Where x_0 is the input sequence of one path and x_1 is that of the other path on the survived state at the l th stage in the trellis diagram. The corresponding pattern reliability output is computed as the path metric difference between the two paths. For the PR channel with AWGN whose

mean is zero and variance is σ_n^2 , this path metric difference is the path log-likelihood ratio (LLR) scaled by $2\sigma_n^2$.

$$L_l = |m_0 - m_1| = 2\sigma_n^2 \log \frac{\Pr(\text{survivor is correct})}{\Pr(\text{survivor is wrong})} \dots\dots\dots(2)$$

where m_0 is the path metric via one path and m_1 is that via the other path on the survived state at the l th stage. Smaller path reliability indicates higher probability of an error event. Computational complexity of the POVA is lower than that of the soft output Viterbi algorithm (SOVA), but memory requirement of the POVA is greater than that of the SOVA.

III. RELIABILITY MATRIX CONSTRUCTION USING THEPOVA OVER PR CHANNELS

For symbol-based soft-decision decoding over PR channels, symbol reliability cannot be obtained from bit reliabilities in the same way as in the AWGN channels due to the correlated nature of bit errors. If symbol reliability is needed only for the most probable symbol like in the GMD algorithm, it is reasonable to take the minimum value among bit reliabilities in a symbol as that symbol's reliability. However, the ASD algorithm requires symbol reliabilities for all possible elements in the finite field. For an (n, k, d) RS code over $GF(q = 2^m)$, when the original codeword is $\mathbf{c} = [c_i]$, a $(q \times n)$ reliability matrix $\mathbf{\Pi} = [\pi_{j,i}]$ is defined as $\pi_{j,i} = \Pr(c_i = a_j | \mathbf{y})$ for $j = 0, \dots, q-1$ and $i = 0, \dots, n-1$, where $a_j = 0$ if $j = 0$, $a_i = \alpha^{j-1}$ otherwise and \mathbf{y} is the channel output.

$$\mathbf{\Pi} = \begin{bmatrix} \Pr(c_0 = a_0 | \mathbf{y}) & \dots & \Pr(c_{n-1} = a_0 | \mathbf{y}) \\ \vdots & \ddots & \vdots \\ \Pr(c_0 = a_{q-1} | \mathbf{y}) & \dots & \Pr(c_{n-1} = a_{q-1} | \mathbf{y}) \end{bmatrix} \dots\dots\dots(3)$$

Generally, the symbol-based BCJR algorithm has been considered as an inevitable choice to build a reliability matrix in spite of its high complexity.

As a replacement of the symbol-based BCJR, we apply the POVA to obtain the reliability matrix for the ASD algorithm. Each error pattern from the POVA captures correlation between erroneous bits resulting from one error event. The likelihood ratio of the discarded path over the survived path can be computed from (2) as follows.

$$R_l = \exp\left(-\frac{L_l}{2\sigma_n^2}\right) = \frac{\Pr(\text{survivor is wrong})}{\Pr(\text{survivor is correct})} \dots\dots\dots(4)$$

When the l th error pattern E_l with reliability L_l covers the symbol location i , let the corresponding error symbol value on i th symbol location be $e_{l,i}$. Then, we assign R_l converted from L_l as the likelihood ratio $\pi_{j,i}^*$ of the symbol value $(r_i + e_{l,i})$ over the received symbol r_i .

$$\pi_{j,i}^* = \frac{\Pr(c_i = r_i + e_{l,i})}{\Pr(c_i = r_i)} = R_l \dots\dots\dots(5)$$

for j such that $a_j = r_i + e_{l,i}$.

For every i , $\pi_{j,i}^*$ is one for the most probable symbol, that is for j such that $r_i = a_j$. The final values, π_j , s, are determined after normalization so that the sum of all entries in each column is one. The proposed reliability matrix construction method is summarized in Algorithm 1, where l_{max} is the maximum length of an error pattern and can be limited by the path memory depth in the implementation of Viterbi algorithm.

Figure 1 shows an example of how to construct a reliability matrix from the outputs of the POVA through the PR channel with channel depth of 3, e.g., the [1 2 2 1] PR channel over GF(23). The variance $\sigma^2 n$ was assumed to be 0.25. Every error pattern ends with one followed by as many zeros as the channel depth as shown in Figure 1, so for each i at least m $e_{l,i}$ s have different values. Including $\pi_{j,i}^* = 1$ for the most probable symbol, at least $(m+1)$ $\pi_{j,i}^*$ s have non-zero values. Consequently, the resulting reliability matrix from Algorithm 1 has at least $(m + 1)$ non-zero elements in each column. This symbol reliability determining process can be merged into POVA by doing symbol alignment of error patterns and carrying out minimum operation inside the POVA. This

can be considered as a generalization of SOVA into symbol unit.

Algorithm 1 Reliability matrix construction from the POVA

```

initialize the reliability matrix  $\Pi$  as all zeros.
for  $i = 0 : n - 1$  do
     $\pi_{j,i} = 1$  for  $j$  such that  $a_j = r_i$ .
end for
for  $l = 0 : nm + l_{max} - 1$  do
    for all symbol locations  $i$ s affected by  $E_l$  do
         $\pi_{j,i} = \max(\pi_{j,i}, R_l)$  for  $j$  such that  $a_j = r_i + e_{l,i}$ .
    end for
end for
normalize  $\Pi$  so that  $\sum_{j=0}^{q-1} \pi_{j,i} = 1$  for  $i = 0, 1, \dots, n - 1$ .
    
```

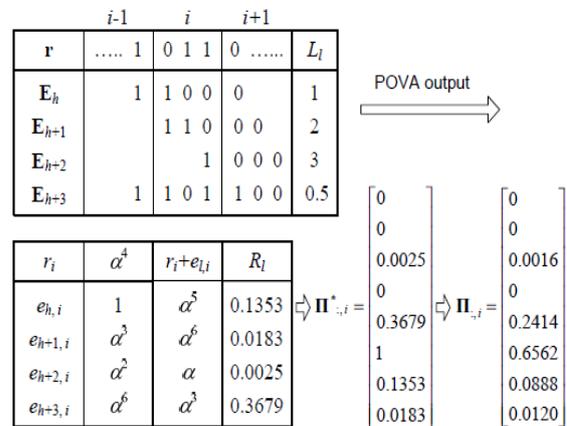


Figure 2. An Example of the Reliability Matrix Construction from the POVA outputs through the PR Channel with Channel Depth of 3 over GF(23).

IV. MODIFICATION OF THE BGMD OVER PR CHANNELS

The BGMD is the GMD-like implementation of the ASD algorithm using bit-level soft information [4]. The BGMD repeats the ASD algorithm while successively modifying the multiplicities of the symbols which include the least reliable its [4]. The benefit of the BGMD is that the algorithm requires only bit-level soft information and generates the multiplicity matrix directly from the soft information without the reliability matrix. This scheme works well over AWGN channels because the single-bit different symbol from the most probable symbol is probably the next most likely symbol. However, over PR

channels, single-bit error symbols are not dominant anymore, so the single-bit different symbol from the most probable symbol may not be the next most probable symbol in many cases. In particular, multi-bit error symbols become more dominant with PR targets for perpendicular magnetic recording channels. Consequently, the efficiency of each repetition of the ASD algorithm decreases over PR channels compared to that over AWGN channels. This is the common problem of bit-based soft-decision decoding algorithms. In order to reflect the property of PR channels where errors happen in patterns, we modify the BGMD by using the POVA. The modified BGMD algorithm is summarized in Algorithm2. The number of erased patterns is parameterized as p .

V. SIMULATION

Before showing simulation results over PR channels, Figure 2 shows the simulation results over AWGN channels for comparison. All simulations were conducted for the (255, 239, 17)RS codes over GF(28). The ASD algorithm with infinite multiplicity provides about 0.55dB coding gain over the conventional hard-decision decoder at codeword error rate (CER) of 10^{-5} . With finite multiplicity, performance gain decreases down to about 0.3dB. The multiplicity matrix is generated with a parameter λ of 4.99 by the method in [7]. Each multiplicity m_j , is computed as

$\lfloor \lambda \pi_{j,i} \rfloor$ which results in maximum multiplicity of 4. The simulation results of the BGMD decoders are also shown in Figure 2. The BGMD with M of 2 shows performance comparable to the ASD algorithm with infinite multiplicity and the BGMD with infinite multiplicity outperforms the ASD with infinite multiplicity.

Algorithm 2 Modification of the BGMD with the POVA

```

initialize the multiplicity matrix  $M = [m_{j,i}]$  as all zeros.
for  $i = 0 : n - 1$  do
     $m_{j,i} = M$  for  $j$  such that  $a_j = r_i$ .
end for
conduct the ASD algorithm with  $M$ .
for  $h = 1 : p$  do
    identify the  $h$ th most probable error pattern  $E_{l_h}$ .
    for all symbol locations  $i$ s affected by  $E_{l_h}$  do
        if  $m_{j,i} = M/2$  for  $j$  such that  $a_j = r_i$  then
            assign zeros to all  $m_{j,i}$ s in that column.
        else
             $m_{j,i} = M/2$  for  $j$ s such that  $a_j = r_i$  or  $r_i + e_{l_h,i}$ .
        end if
    end for
    conduct the ASD algorithm with  $M$ .
end for
choose the most probable codeword out of candidate codewords.
    
```

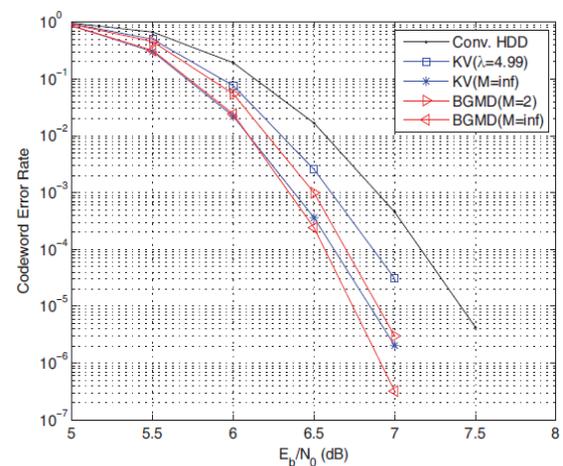


Figure 3. Simulation results of the ASD algorithm and the BGMD algorithm for the (255, 239, 17) RS code over AWGN channel.

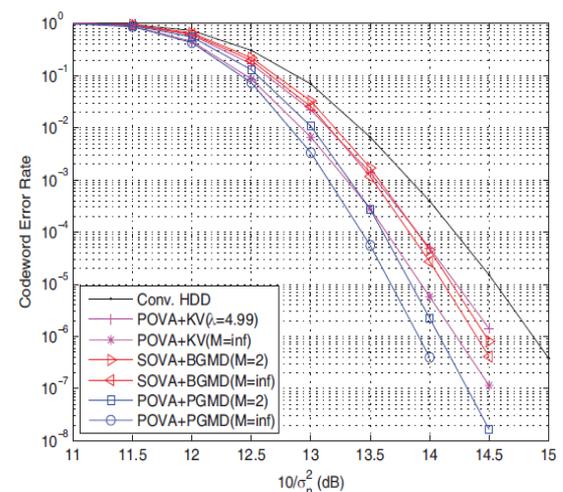


Figure 4. Simulation Results of the ASD Algorithm with POVA and the Modified BGMD (PGMD) for the (255,239) RS code over the [1 2 2 1] PR Channel.

We simulated the ASD algorithm over the [1 2 2 1] PR channel with AWGN whose mean is zero and variance is $\sigma^2 n$ for the same RS code. The ASD algorithm employs the proposed pattern-based reliability matrix construction algorithm using the POVA shown in Algorithm 1. Figure 3 shows the simulation results of the proposed algorithm over the PR channel. The ASD algorithm with infinite multiplicity provides about 0.6dB coding gain and the ASD with finite multiplicity provides about 0.35dB gain over the conventional hard-decision decoder. It can be seen that by using the POVA to build reliability matrices, the performance gains of the ASD algorithm with finite multiplicity and infinite multiplicity over the PR channel are greater than those over the AWGN channel, respectively.

The simulation results of the modified BGMD with p of 16 using the POVA (denoted by PGMD) are also shown in Figure 3. The original BGMD using the SOVA, which conducts the same number of ASDs, was also simulated for comparison. The modified BGMD shows even better performance gain over the PR channel compared to the BGMD over the AWGN channel. The modified BGMD with infinite multiplicity provides about 0.9dB coding gain and that with finite multiplicity provides about 0.7dB gain over the conventional hard-decision decoder. On the other hand, for both finite multiplicity and infinite multiplicity, the performance gains of the original BGMD over the PR channel decrease significantly compared to those over the AWGN channel shown in Figure 2.

V. RESULTS

Convolution Encoder: These are used to apply the encoding scheme for sequential circuits of the circuits. Encoders are here used to apply the bit sequences for

protecting the input message data. here we are applying 3 bit convolution encoders with two bits as input bit sequences.

When valid=1, in accordance to the input clock, the input bits would generate the symbols (convolution bits) as per the equations given in the diagram of your paper. Diagram would be as shown below.



Figure 5: Convolution encoder output

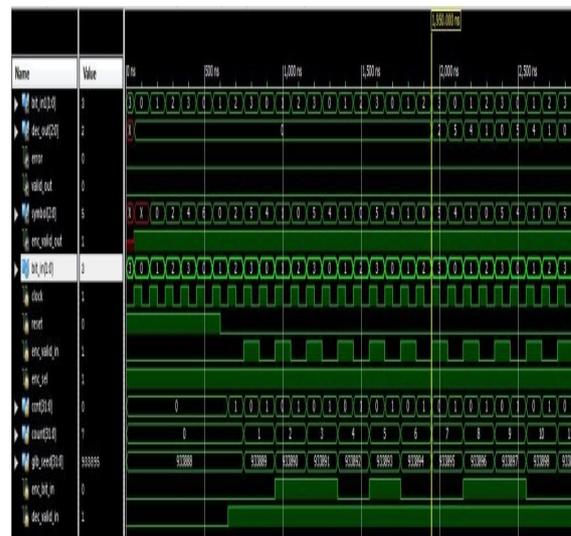


Figure 6: Final Output

VI. CONCLUSIONS

We showed that the POVA can be effectively applied to the ASD algorithm over PR channels. The POVA provides the probable error patterns with corresponding pattern reliabilities and that information can

be transformed into symbol reliabilities after some processing to build a reliability matrix. In a similar way, the POVA can be also applied to the GMD type extension of the ASD algorithm by modifying the BGMD algorithm. The simulation results showed that the POVA can successfully replace the symbol-based BCJR algorithm over PR channels providing attractive performance gain.

REFERENCES:

- [1] R. Koetter and A. Vardy, "Algebraic soft-decision decoding of Reed-Solomon codes," *IEEE Trans. Inf. Theory*, vol. 49, no. 11, pp. 2809–2825, 2003.
- [2] V. Guruswami and M. Sudan, "Improved decoding of Reed-Solomon and algebraic-geometry codes," *IEEE Trans. Inf. Theory*, vol. 45, pp. 1757–1767, 1999.
- [3] P. Hoeher, "Optimal sub block-by-sub block detection," *IEEE Trans. Commun.*, vol. 43, no. 234, pp. 714–717, 1995.
- [4] J. Jiang and K. R. Narayanan, "Algebraic soft-decision decoding of Reed-Solomon codes using bit-level soft information," *IEEE Trans. Inf. Theory*, vol. 54, no. 9, pp. 3907–3928, Sep. 2008.
- [5] G. D. Forney, "Generalized minimum distance decoding," *IEEE Trans. Inf. Theory*, vol. 12, no. 2, pp. 125–131, 1966.
- [6] S.-W. Lee and B. V. K. Vijaya Kumar, "Pattern-flipping Chase-type decoders with error pattern extracting Viterbi algorithm," to appear in *IEEE J. Sel. Areas Commun.*, 2010.
- [7] W. J. Gross, F. R. Kschischang, R. Koetter, and P. G. Gulak, "Simulation results for algebraic soft-decision decoding of Reed-Solomon codes," *IEEE Trans. Commun.*, vol. 54, pp. 1224–1234, July 2003.

* * * * *