

Decentralized PID Controller Design for TITO Processes for Uncertain System

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ABSTRACT

In this paper, a design method of decentralized Proportional–integral–derivative PID) controller for two inputs two-output processes Based on pre-defined reference Transfer function is proposed. An ideal decoupler is used to reduce the interaction among system variables. Free structure higher order diagonal controllers are computed for each Decoupled subsystem by specifying closed-loop response in Terms of dominant reference second order transfer function. Further, to obtain controllers in PID structure, the higher order diagonal controllers are truncated into first three terms of Maclaurin series. The stability of resulting PID controller is investigated. Two benchmark examples are illustrated to Show the effectiveness of the proposed controller. *Experimentation is performed on interacting* coupled tank process to demonstrate the applicability in real life applications.

Keyword:— Multivariable systems, Ideal decoupler, Decentralized controller, Parameter uncertainty, Experimental Application.

1. INTRODUCTION

Many of the industrial processes posses multiple input multiple output (MIMO) S. S. Sankeshwari Professor & Head of the Department Department of Electrical, Electronics & Power Engg. MBES's College of Engineering, Dr Babasaheb Ambedkar Marathwada University, Ambajogai, (M.H.) [INDIA]

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dynamics. The design of controller for MIMO systems is dicult due to process and loop Interactions and hence poses a challenging control task. The controller design methods for MIMO systems reported in the literature can be broadly classified as centralized (full Structure) and decentralized (diagonal) controllers. Dierent approaches for designing Centralized controllers are reported in the literature including recent publications such As [1,2]. In these approaches, interactions are reduced using full structure controller And the loop controllers interact with each other. Hence, the tuning for individual Loop controller done independently which cannot be complicates the design procedure.

In process control industries, multi loop single input single output (SISO) controllers are often used to control plants having MIMO dynamics [3]. The most attractive advantages of such methods are structural simplicity and the easiness to handle loop failure. However, due to interaction the multi loop SISO controllers encounter more difficulties than that of a single loop which may result in unacceptable performance and has became an open research topic for the years [4]. Many design methods have been reported in literature for multi loop SISO controller design.

Some of them include detuning methods [4, 5], sequential loop closing (SLC), [68] and independent methods[9, 10]. In detuning approach, the o diagonal elements in process transfer function matrix are ignored and the diagonal controllers are tuned based on single loop controller design approach. The diagonal controllers are then detuned by the detuning factor obtained from the interaction measure like relative gain array (RGA). A well known Detuning method is the biggest log modulus tuning (BLT) given in [4]. In this method,

Each individual controller is first designed for corresponding diagonal element based on Conventional tuning rules [11]. As mentioned, most commonly used controllers in various multi-variable process industries are proportional integral derivative (PID) type, despite the advanced control strategies like model predictive control. PID controllers the simplicity and yet most e cient solution to many real-world control problems. The functionality of three-term PID controllers deals with wrapper treatment of both transient and steady state responses. It is well known that the aim of a control system design is to develop a control law which gives desired response of a given process.

The desired response can be achieved with a closed-loop control system, where the controller determines the input signal to the process by using the measurement of the output or the feedback signal. Feedback control is actually essential to keep the process variable close to the desired value in spite of disturbances and variations of the process dynamics. The development of feedback control methodologies has a tremendous impact in many different fields of the engineering.



Figure 1: Typical feedback control system

The typical feedback control system is represented in the feedback systems include process, controller, actuator(s), and senso(s). The overall control system performance depends on the proper choice of each component. From the view of controller design, the actuator and sensor dynamics are often neglected (only steady state gains and the saturation limits of the actuator have to be considered).

II. SYSTEM DESCRIPTION AND CONTROLLER DESIGN

The structure of multi-variable feedback control system with decoupler is shown in Figure 2.



Figure 2: Block diagram of closed-loop System with Decoupler

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In Figure 2, r1. ldots rn and y1, ldots, yn are reference inputs and outputs of the system respectively. u1. ldots un and v1. ldots vn are controllers outputs and decoupler outputs respectively. The controller design method is based on reference Transfer function of typical second order systems. An ideal decoupler is used to obtain SISO subsystems [20]. The structure of MIMO system and ideal decoupler is given by

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) & \cdots & G_{1n}(s) \\ G_{21}(s) & G_{22}(s) & \cdots & G_{2n}(s) \\ \vdots & \vdots & \vdots & \vdots \\ G_{n1}(s) & G_{n2}(s) & \cdots & G_{nn}(s) \end{bmatrix}$$

and

$$D(s) = \begin{bmatrix} D_{11}(s) & D_{12}(s) & \cdots & D_{1n}(s) \\ D_{21}(s) & D_{22}(s) & \cdots & D_{2n}(s) \\ \vdots & \vdots & \vdots & \vdots \\ D_{n1}(s) & D_{n2}(s) & \cdots & D_{nn}(s) \end{bmatrix}$$

Respectively. The equivalent multi loop SISO

structure is Q(s) = G(s)D(s),

Where,

$$Q(s) = \begin{bmatrix} Q_{11}(s) & 0 & \cdots & 0 \\ 0 & Q_{22}(s) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & Q_{nn}(s) \end{bmatrix}$$

Decoupler in Eq. (3) can be represented as,

$$D(s) = Adj[G(s)]K(s)$$

Where K(s) is a diagonal matrix. The elements *kii* (*s*) are obtained such that common pole-zero, common dead time and smallest gain from *i*th column of Adj [G(s)] are removed

$$Q_{ii}(s) = \frac{b_{0ii}s^{m_{ii}} + b_{1ii}s^{(m-1)_{ii}} + \dots + b_{mii}}{s^{n_{ii}} + a_{1ii}s^{(n-1)_{ii}} + \dots + a_{nii}}, \ i = 1, 2$$

and their inverse is included in *kii* (s). The dead time term of Q(s) in Eq. (3) can be approximated in different ways. In this work, first-order Taylor series approximation, that is $e-\tau d \ s = 1 \ \tau d \ s+1$ is used for dead time term. The decoupled subsystems in Eq. (3) with delay approximation can be represented in general form as

Where $m \leq n$, *aii's* and *bii's* are polynomial coefficients. Also, it is presumed that all decoupled subsystems in *Qii* (s) do not contain any right half plane (RHP) zero. The closed loop transfer function *Tii* (s) between output and set-point of *i*th loop is given by

$$T_{ii}(s) = \frac{Q_{ii}(s)G_{cii}}{1 + Q_{ii}(s)G_{cii}}, \ i = 1, 2$$

Where, Gcii is the controller for each diagonal subsystem in Qii (s). According to the IMC theory [29], the closed-loop transfer function Tfii (s) of the system is

$$T_{fii}(s) = \frac{1}{(\tau_i s + 1)^{q_i}}, \ i = 1, 2$$

where, τi indicates IMC filter constant, the term *qi* is the relative order of the numerator and denominator in *Qii* (s) and chosen equal to or greater than nii - mii. The reference closed- loop trajectory given in Eq. (7) includes repeated real poles. The expected response of system is critically damped which depends on two unknown parameters, τi and qi? The selection of design parameter τi in IMC is a key decision and several guidelines have been published in the literature [29]. However, different guidelines generate different performance for the same process. To get liberty from such limitations and to place

the dominant poles, without loss of generality, the Desired reference transfer function can be selected as

$$T_{dii}(s) = \frac{1}{(\lambda_i s + 1)^{q_i - 2}} \frac{\omega_{ni}^2}{s^2 + 2\zeta_i \omega_{ni} s + \omega_{ni}^2}$$

Where, ζi and ωni indicate damping factor and undamped natural frequency respectively. The term $\zeta i \in (0.7, 0.99)$ which is selected from peak overshoot and ωni is calculated from expected settling time ts using relation $\omega ni = 4/(\zeta i tsi)$. The closed-loop poles can be placed at desired location by assigning value of ζi and $\omega n i$. The term λi in Eq. (8) is chosen such that (qi - 2) repeated poles are far away from dominant pole pair to make them nondominant. Equating Eqs. (6) And (8), the controller Gcii (s) can be written as

$$G_{cii}(s) = \frac{1}{s}g_{ii}(s)$$

where,

$$g_{ii}(s) = \frac{s}{Q_{ii}(s)} \frac{\omega_{ni}^2}{(1 + \lambda_i s)^{q_i - 2} (s^2 + 2\zeta_i \omega_{ni} s + \omega_{ni}^2) - \omega_{ni}^2}$$

Note that in Eq. (9) there is no nonminimum phase part of Qii (s) and hence the controller has neither causality nor stability issue. Multi-loop controllers obtained in Eq. (9) are of non PID structure. In view of the fact that most common controllers used in process industries are of PI/PID form, to obtain a multi-loop PID can be controller structure, Eq. (9) approximated most closely using Maclaurin series expansion as

$$G_{cgii}(s) = \frac{1}{s} \left[g_{ii}(0) + g_{ii}'(0)s + \frac{g_{ii}''(0)}{2!}s^2 + \cdots \right]$$

The standard form of PID controller is

$$G_{cii}(s) = K_{Pii} + \frac{K_{Iii}}{s} + K_{Dii}s$$
$$= \frac{1}{s}(K_{Iii} + sK_{Pii} + s^2K_{Dii})$$

where, KPii, *KI ii* and *KDii* represent proportional, integral and derivative gain respectively. Hence, using first three terms of Eq. (10) a standard PID controller is,

KPii = *gii* (0); *KI ii* = gii (0); *KDii* = *gii* (0)

III. STABILITY ANALYSIS

From a well known generalized Nyquist stability theorem presented in [30] assume that $G-\delta$ system structure shown in Figure 2 is stable where G(s), $\delta(s)$ are the nominal system and the perturbation respectively. Consider that, if δ is the convex set of perturbation and δ_{-} is an allowed perturbation, then $_{-\delta_{-}}$ is small change in δ_{-} where _ is any real scalar suchthat $|_{-}| \leq 1$. Then system shown in Figure 2 is stable for all δ if and only if any one of the following equivalent conditions is satisfied:

- 1. Nyquist plot of det $(I G\delta(s))$ does not encircle the origin, $\forall \delta$, i.e. det $(I - G\delta(j\omega)) = 0$, $\forall \omega$, $\forall \delta$
- 2. $\tau i G \delta(j \omega) = 1, \forall i, \forall, \forall \delta$
- 3. $\rho G\delta(j\omega) < 1, \forall \omega, \forall \delta, where \rho is$ spectral *radius*.
- 4. $\max \delta \rho G \delta(j \omega) < l, \forall \omega$

Now a nominal (2×2) multiloop control structure from Figure 1 is nominally stable if and only if

1.
$$\frac{G_{C11}}{1 + G_{11}G_{C11}}$$
 and $\frac{G_{C22}}{1 + G_{22}G_{C22}}$ are stable
2. $\rho\left(\begin{bmatrix} 0 & \frac{G_{12}G_{C11}}{1 + G_{11}G_{C11}}\\ \frac{G_{21}G_{C22}}{1 + G_{22}G_{C22}} & 0 \end{bmatrix}\right) < 1, \quad \forall \omega$

Actuator uncertainties, and measurement uncertainties due to sensor are often encountered is of lump multiple sources of a multiplicative form uncertainty. In this

work to identify the closed-loop system robust stability in the presence of such uncertainties, Eqs. (13) and (14) are used to evaluate the closed loop system robust stability.

$$\rho \left(G_C (I + GG_C)^{-1} G \delta_I \right) < 1, \quad \forall \omega$$
$$\rho \left(GG_C (I + GG_C)^{-1} G \delta_O \right) < 1, \quad \forall \alpha$$



Figure 3: General G— δ Structure

Where, δI and δO are stable process multiplicative input and output uncertainties respectively. Hence, for a specified bound of δI and δO , control system robust stability can be Evaluated by observing the magnitude plots of the left sides of Eqs. (13) And (14) with $\omega \in [0, +\infty]$, which should fall below unity. This can be conveniently performed by control software packages, like MATLAB robust control toolbox. This is demonstrated in simulation

Example 1.

$$\begin{split} G_{ch1} &= \frac{352.6s^6 + 497.8s^5 + 168s^4 + 24.64s^3 + 1.805s^2 + 0.065s + 0.00092}{s(s^5 + 101.2s^4 + 122.1s^3 + 22.98s^2 + 1.579s + 0.034)} \\ G_{ch2} &= \frac{-180.5s^6 - 128.7s^5 - 33.04s^4 - 4.133s^3 - 0.27\natural 6s^2 - 0.0092s - 0.00012}{s(s^5 + 100.5s^4 + 52.63s^3 + 7.98s^2 + 0.48s + 0.0094)} \end{split}$$

IV. SIMULATION EXAMPLES

Example: Wood–Berry (WB) binary distillation column process

Wood and Berry introduced the transfer function model of a pilot-scale distillation column, which consists of an eight tray plus reboiler separating methanol and water [31]. The Wood–Berry binary distillation column process is a multivariable system that has been studied extensively by many researchers [17,22]. The process has the transfer function matrix as

$$G(s) = \begin{pmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{pmatrix}.$$

The decoupler determined using Eq. (4) is

$$D(s) = \begin{pmatrix} \frac{2.94}{14.4s+1} & \frac{1.477e^{-2s}}{21s+1} \\ \frac{e^{-4s}}{10.9s+1} & \frac{1}{16.7s+1} \end{pmatrix}.$$

The Decoupled subsystems are

$$Q_{11}(s) = \frac{37.63}{(16.7s+1)(14.4s+1)(s+1)} \\ -\frac{18.9}{(21s+1)(10.9s+1)(7s+1)},$$
$$Q_{12}(s) = Q_{21}(s) = 0,$$

and

$$Q_{22}(s) = \frac{9.75}{(10.9s+1)(21s+1)(9s+1)} - \frac{19.4}{(14.4s+1)(16.7s+1)(3s+1)}$$

Using Eq. (8), with $\lambda 1 = \lambda 2 = 0.01$, q1 = q2 = 3, $[\zeta 1, \zeta 2] = [0.7, 0.8]$ and settling time [ts1, ts2] = [8, 25], the resulting higher order free controllers *Gch*1 and *Gch*2 are obtained and given in Eqs. (15) and (16).

Higher order free structure controllers *Gch*1 and *Gch*2 are converted into PID structure using Eq. (12). The resulting PID controller parameters are given in Table 1.The performance of proposed method is compared with Maghade and Patre [22], NDT [17]method and Shen et al. [32] methods. In this example Maghade and Patre

have used filters 15.5252s+1 for first and 15.1499s+1 for second PID controller.

The desired location of closed loop poles with higher order controllers and PID controllers are given in Table 2. Output response of the system with higher order free controller and PID controller is shown in Figure 3. It is observed that response is nearly same for both the controllers. For nominal model, *tsi* settling time in minute and *I SEi* integral square error are given in Table 3. In this example, NDT and Maghade methods are based on decoupler strategy

$$D_{NDT}(s) = \left(\frac{1}{\frac{0.34(14.4s+1)e^{-4s}}{10.9s+1}} \frac{1.477(16.7s+1)e^{-2s}}{21s+1}}{1}\right)$$

and Shen et al. [32] is based on adjoint transfer matrix decoupling strategy.

$$D(s)_{Shen} = \begin{bmatrix} \frac{-19.4}{14.4s+1} & \frac{18.9e^{-2s}}{12s+1} \\ \frac{-6.6}{10.9s+1} & \frac{12.8}{16.7s+1} \end{bmatrix}$$

The unit step change is applied to the first set-point for time $t \ge 0$ keeping second set-point zero for $0 \le t \le 120$ and changed it to unity for time $t \ge 120$. The output responses given by various controllers are show in Figure 4. From Figure 4; it is clear that Maghade and Patre's method Produce oscillatory responses while Y. Shen's method gives slow response.NDT method is comparable with the proposed controller, however the interaction given by proposed compared to other method less is controllers.

Table 1: Tuning Parameter for Wood-BerryExample

Method	[KP11 Kl11, KD11]	[KP22 Kl22, KD22]	
Proposed	[0.657 0.027 4.301]	[-0.312 - 0.013 2.175]	
Maghade and Patre [22]	[0.9733 0.0881 2.6887]	[-0.3134 0.0304 -0.807]	
Shen et al. [32]	[-0.06187 -0.002542 -0.3421]	[-0.02062 -0.00085 - 0.1141]	
NDT[17]	[0.41 0.074 -]	[-0.120 -0.024 -]	

Table 2: Location of Desired close loop
poles for Example 1

Subsystem	Desired	Higher order Controller (n = 6)	PID Controller
Q11	$-0.5 \pm 0.51i$	-0.5 ± 0.51i	$-0.52 \pm 0.63i$
Q22	$\textbf{-0.16} \pm 0.12 i$	$-0.16\pm0.12i$	$-0.19 \pm 0.15i$



Figure 4: Output responses of Example 1 for higher order and PID controller a Response of y1 Example 1, b Responses of y2 of Example 1



Figure 5: Responses of Example 1. a y1 response to unit step in r1-at t=0 and r2 - at t = 120. b y2 response to unit step in r1 - at t=0 and r2 - at t = 120

Table 3: Performance indices for National and ± 20% uncertainty in model for the WB column

Method	Normal t _{s1} , t _{s2} , ISE ₁ , ISE ₂ ,	+20% parametric uncertainty t _{s1} , t _{s2} , ISE ₁ , ISE ₂ ,	-20% parametric uncertainty t _{s1} , t _{s2} , ISE ₁ , ISE ₂ ,
Proposed	[13.5, 25, 1.82,5.62]	[12, 20, 1.82,5.78]	[18.7, 40, 1.71, 3.81]
Maghade	[35, 22, 2.73, 4.84]	[50, 60, 4.1, 7.98]	[7.8, 150, 2.76, 18.02]
NDT	[23, 30, 2.35, 5.32]	[17, 50, 3.01, 8.38]	[10.8, 30, 2.28, 4.97]
Y.Shen et. at	[20, 28, 2.32, 6.92]	[15, 62, 2.67, 8.16]	[29.6, 34, 2.16, 6.15]



Figure 6: Magnitude Plots of Special Radius for Example 1

To show the multi-loop control system stability of the proposed method, assume process multiplicative that the input uncertainty (δI) in loop-1 and loop-2 is (s + (0.3)/(s + 1). It means that inputs supplied by the corresponding actuators to the process are increased up to 100% uncertainty at high frequencies and 30% uncertainty in the low frequency range. Further, assume that the process multiplicative output Uncertainty (δO) in loop-1 and loop-2 is (s + 0.2)/(2s)+1). It means measurements provided by the corresponding sensors decrease with up to 20% uncertainty at high frequencies and with almost 20% uncertainty in the lowfrequency range. Figure 6 shows the magnitude plots of spectral radius in terms of the assumed nominal, δI and δO for proposed control system robust stability. It is observed that, δI and δO falls below the unity. To investigate the robustness in case of process parametric uncertainty, all four time delays, time constants and gains are changed by} The closed loop 20%. responses for different control schemes are shown in Figure 6 and quantitative performance indices are tabulated in Table 3. It can be concluded that performance of the proposed controller is robust with less interaction among the variables.

V. CONCLUSION

This paper proposes a method to design decentralized PID controller for TITO process based on the reference dominant transfer function. The TITO process is decoupled through an ideal decoupler and controllers are obtained from decoupled subsystems. The Maclaurin series expansion is used to determine parameters of the PID controller.

The robust stability of the resulting PID controller is investigated. To show the effectiveness proposed technique. of simulation Examples are included and the performance of the proposed controller is compared with the prevalent controllers. To show the robustness of the proposed controller,} 20% parametric uncertainty is added simultaneously, in all time delays, time constants and gains. Simulation results illustrate that this design method provides good performance with less interaction and has robust performance under the effect of Parametric uncertainty. То show the applicability of the proposed controller, experimentation is performed on real life interacting coupled tank liquid level system. The experimental result shows that the controller outputs are smooth which drives the pumps smoothly even under the effect of Disturbance. Such type of smooth behavior of controller is expected which gives long life to final control elements without wear and tear.

VI. FUTURE SCOPE

The presented method in the report were restricted to TITO systems, some directions in which the present work can be extended are as follows:

The MIMO controller design method is discussed in thesis, has straight forward Extension to design multi-loop SISO controllers for higher dimensional multi-Variable systems. The performance of the MIMO controller design method can be improved by appropriate selection of the detuning parameter which is based on systems properties. The proposed design

methods can be extended for on- line autotuning of a controller.

The method can be extended to the systems those are having more than two inputs and two outputs. This work can be extended to discrete time LTI system models to design discrete multi-variable controllers.

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