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Intuitionistic Fuzzy Semi Distributive Lattices

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ABSTRACT

The concept of lattice plays a significant role in Mathematics. The notion of intuitionistic fuzzy set was introduced by Atanassov as a generalization of the fuzzy sets. Intuitionistic fuzzy join semi distributive lattice and intuitionistic fuzzy meet semi distributive lattice are the two types of binary operations of intuitionistic fuzzy semi distributive lattices are proposed . New notions and definitions are proposed of intuitionistic fuzzy distributive lattices.

Keywords:— *Intuitionistic fuzzy distributive lattice, Intuitionistic fuzzy join semi distributive lattice, intuitionistic fuzzy meet semi distributive lattice*

I. INTRODUCTION

In Mathematics since 18th century onwards lattices have been used. In lattices modular lattice is one of the most important types. The theory of fuzzy sets proposed by Lotfi A Zadeh in 1965 has achieved a great success in various fields. Intuitionistic fuzzy sets which are very effective to deal with haziness presented by K. Atanassov [1] with the research of fuzzy sets in 1986. The concept of intuitionistic fuzzy sets is a generalization of fuzzy sets. Bustince and Burillo proposed the concept of intuitionistic fuzzy relations and investigated some of its properties and Yon and Kim introduced the notion of intuitionistic fuzzy sub lattices, filters and ideals. N. Ajmal and K. V. Thomas [6] initiated such types of study in the year 1994. It was later independently established by N. Ajmal that the set of all fuzzy normal subgroups of a group constitute a sub lattice of the lattice of all subgroups and is modular lattice. S. Nanda [5] proposed the notion of fuzzy lattice using the concept of fuzzy partial ordering. The concept of fuzzy lattice introduced by N. Ajmal, S. Nanda and Wilcox. L. R [5,6,7] explained modularity in the theory of lattices. G. Gratzer [3], G. H. Bar Dalo, E. Rodrigues and M. Stern [2] explained semi modular Lattices. In this paper Intuitionistic fuzzy join semi distributive lattice and intuitionistic fuzzy meet semi distributive lattice are the two types of binary operations of intuitionistic fuzzy semi distributive lattices are proposed. New notions and definitions are proposed of intuitionistic fuzzy distributive lattices and also some examples are given.

II. PRELIMINARIES

2.1. Definition: [K. Atanassov, 1986]: Let X be a nonempty set. An intuitionistic fuzzy set A is an object having the form: A = {x, μ_A (x),ν_A (x); x ∈ X} where the function μ_A: X→[0,1] defines the degree of membership andν_A: X→[0,1] defines the degree of non-membership of each element x∈X to the set

A and $0 \leq \mu_A(x)+\nu_A(x) \leq 1$; for every x∈X. For convenience we can use $A = (\mu_A,\nu_A)$.

2.2. Definition: [Kul Hur, S. Y. Jang, Y. B. Jun, 2005] Let X and Y be two sets. An intuitionistic fuzzy relation R from X to Y is an intuitionistic fuzzy set of X × Y characterized by the membership function μ_R in X × Y and non-membership function $\nu_{(R)}$ in X × Y that is, R = {(x, y), μ_R (x, y), $\nu_{(R)}$ (x, y):x∈X, y ∈Y} an intuitionistic fuzzy relation R from X to Y will be denoted by R(X × Y).

2.3. Definition: [KulHur, S. Y. Jang, Y. B. Jun, 2005] An intuitionistic fuzzy relation R is

1. Reflexive if for every x∈X;$\mu_R(x, x) = 1$ and $\nu_R(x, x) = 0$
2. Anti symmetric if for every (x, y) ∈ X × X ;
   $\mu_R(x, y) > 0$ and $\mu_R(y, x) > 0$ or $\nu_R(x, y) < 1$ and $\nu_R(y, x)<1 \Rightarrow x = y$
3. Transitive ifR∘R⊂R , where ∘ is max-min and min-max composition, that is $\mu_R(x, z) \geq [\max]_y [\min\{\mu_R(x, y),\mu_R(x, y)\}]$and

$$\nu_R(x,z) \leq \min_y[\max\{\nu_R(x,y),\nu_R(y,z)\}]$$

2.4. Definition: [K. V. Thomas, N. Ajmal] Let L be a lattice and A = {⟨x, μ_A (x), ν_A (x) ⟩/x∈L} be an intuitionistic fuzzy set of L. ThenAis called an intuitionistic fuzzy sublattice of L if the following conditions are satisfied

1. $\mu_A(x \vee y) \geq \min\{\mu_A(x),\nu_A(y)\}$
2. $\mu_A(x \wedge y) \geq \min\{\mu_A(x),\nu_A(y)\}$
3. $\nu_A(x \vee y) \leq \max\{\nu_A(x),\nu_A(y)\}$
4. $\nu_A(x \wedge y) \leq \max\{\nu_A(x),\nu_A(y)\}$ for all x, y∈L.

The set of all intuitionistic fuzzy sublattice is denoted by IFL.

2.5. Definition: [K. V. Thomas, N.Ajmal] A fuzzy subset μ of L is called a fuzzy sublattice of L if

1. $\mu(x \vee y) \geq \min\{(\mu(x), \mu(y)\}$
2. $\mu(x \wedge y) \geq \min\{(\mu(x), \mu(y)\}$, for all x, y ∈L.

2.6. Definition: [Wilcox. L. R] Let L be a fuzzy lattice and μ(a), μ(b) in L. Thus (μ(a), μ(b)) is called a fuzzy modular pair if

$\mu(c)\vee\mu(a \wedge b) = \mu(c \vee a) \wedge \mu(b)$, for all $\mu(c) \leq \mu(b)$ in L.

That is $\mu(c) \vee [\mu(a) \wedge \mu(c \vee b)] = \mu(c \vee a) \wedge \mu(c \vee b)$, for all μ(c) in L.

2.7. Definition: A Fuzzy lattice L is called a Fuzzy join-semi distributive if $\mu(a \vee b) = \mu(a \vee c)$ then $\mu(a \vee b) = \mu(a) \vee \mu(b \vee c)$ for all μ(a), μ(b), μ(c)∈L

2.8. Definition: A Fuzzy lattice L is called a Fuzzy meet-semi distributive if$\mu(a \wedge b) = \mu(a \wedge c)$ then $\mu(a \wedge b) = \mu(a) \wedge \mu(b \vee c)$ for all μ(a), μ(b), μ(c)∈L

2.9. Definition: A Fuzzy lattice L is called a Fuzzy distributive if $\mu(a) \vee \mu(b \wedge c) = \mu(a \vee b) \wedge \mu(a \vee c)$ for all μ(a), μ(b), μ(c)∈L

III. INTUITIONISTIC FUZZY JOIN SEMI DISTRIBUTIVE LATTICE

3.1. Definition: Let ⟨L, μ_L, ν_L ⟩ is an IFL and is called an intuitionistic fuzzy join semi distributive lattice if [IFJSDL]

$\mu_L(a \vee b) = \mu_L(a) \vee \mu_L(b \wedge c)$

Whenever $\mu_L(a \vee b) = \mu_L(a \vee c)$

$\nu_L(a \vee b) = \nu_L(a) \vee \nu_L(b \vee c)$

Whenever $\nu_L(a \vee b) = \nu_L(a \vee c)$

for all $(\langle \mu_L(a), \nu_L(a) \rangle, \langle \mu_L(b), \nu_L(b) \rangle, \langle \mu_L(c), \nu_L(c) \rangle) \in$ IFL.

3.2. Theorem: Every intuitionistic fuzzy join semi distributive lattice [IFJSDL] is an intuitionistic fuzzy lattice [IFL] and the converse is not true

Proof: Given $\langle L, \mu_L, \nu_L \rangle$ is an IFJSDL

$\mu_L(a \vee b) = \mu_L(a) \vee \mu_L(b \wedge c)$ and $\nu_L(a \vee b) = \nu_L(a) \vee \nu_L(b \vee c)$

For all$(\langle \mu_L(a), \nu_L(a) \rangle, \langle \mu_L(b), \nu_L(b) \rangle, \langle \mu_L(c), \nu_L(c) \rangle) \in$ IFL

To prove that $\langle L, \mu_L, \nu_L \rangle$ is an IFL

That is to prove $\mu_L(a \vee b) = \mu_L(a \vee c)$ and $\nu_L(a \vee b) = \nu_L(a \vee c)$

for all $(\langle \mu_L(a), \nu_L(a) \rangle, \langle \mu_L(b), \nu_L(b) \rangle, \langle \mu_L(c), \nu_L(c) \rangle) \in$ IFL.

Then $\mu_L(a \vee b) = \mu_L(a) \vee \mu_L(b \wedge c)$

$\geq \min\{\mu_L(a), \mu_L(b \wedge c)\}$

$\geq \min\{\mu_L(a), \min\{\mu_L(b), \mu_L(c)\}\}$

$\geq \min\{\mu_L(a), \mu_L(c \wedge b)\}$

$= \mu_L(a) \vee \mu_L(c \wedge b)$

$= \mu_L(a \vee c)$

Also $\nu_L(a \vee b) = \nu_L(a) \vee \nu_L(b \vee c)$

$\leq \max\{\nu_L(a), \nu_L(b \vee c)\}$

$\leq \max\{\nu_L(a), \max\{\nu_L(c), \nu_L(b)\}\}$

$\leq \max\{\nu_L(a), \nu_L(c \vee b)\}$

$= \nu_L(a) \vee \nu_L(c \vee b)$

$= \nu_L(a \vee c)$

Hence $\langle L, \mu_L, \nu_L \rangle$ is an IFL.

The converse need not be true, that is every IFL need not be an IFJSDL.

We shall verify it by the following example

Consider an IFL D_5 of the following figure

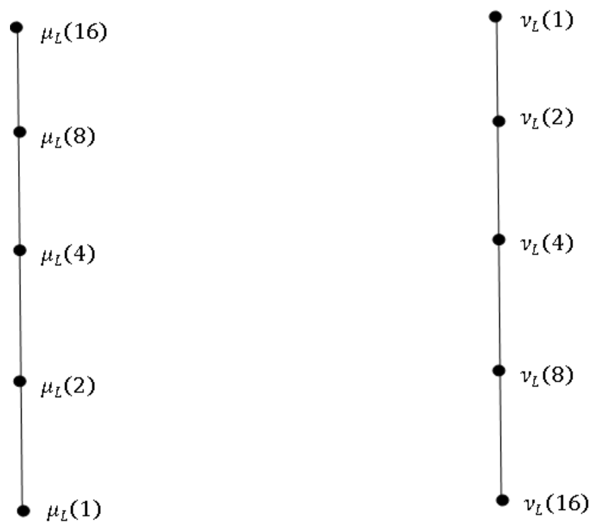

Figure (1). Intuitionistic fuzzy lattice of

Consider (a, b) = (1, 2)

Put c = 4, 8, 16

Equations becomes

$\mu_L(a \vee b) = \mu_L(a) \vee \mu_L(b \wedge c)$............(1)

$\nu_L(a \vee b) = \nu_L(a) \vee \nu_L(b \vee c)$.............(2)

Put c = 4 in equation (1)

$\mu_L(1 \vee 2) \geq \min\{\mu_L(1), \mu_L(2 \wedge 4)\}$

$\geq \min\{\mu_L(1), \mu_L(2)\}$

$\mu_L(2) = \mu_L(2)$

Put c = 4 in equation (2)

$\nu_L(1 \vee 2) \leq \max\{\nu_L(1), \nu_L(1 \vee 4)\}$

$\leq \max\{\nu_L(1), \nu_L(1)\}$

$\nu_L(1) = \nu_L(1)$

Put c = 8 in equation (1)

$\mu_L(1 \vee 2) \geq \min\{\mu_L(1), \mu_L(2 \wedge 8)\}$

$\geq \min\{\mu_L(1), \mu_L(2)\}$

$\mu_L(2) = \mu_L(2)$

Put c = 8 in equation (2)

$\nu_L(1 \vee 2) \leq \max\{\nu_L(1), \nu_L(1 \vee 8)\}$

$\leq \max\{\nu_L(1), \nu_L(1)\}$

$\nu_L(1) = \nu_L(1)$

Put c = 16 in equation (1)

$\mu_L(1 \vee 2) \geq \min\{\mu_L(1), \mu_L(2 \wedge 16)\}$

$\geq \min\{\mu_L(1), \mu_L(2)\}$

$\mu_L(2) = \mu_L(2)$

Put c = 16 in equation (2)

$\nu_L(1 \vee 2) \leq \max\{\nu_L(1), \nu_L(1 \vee 16)\}$

$\leq \max\{\nu_L(1), \nu_L(1)\}$

$\nu_L(1) = \nu_L(1)$

$(<\mu_L(1), \nu_L(1)>, <\mu_L(2), \nu_L(2)>)$ is an IFJSDL.

Consider (a, b) = (2,4)

Put c = 1, 8, 16

Put c = 1 in equation (1)

$\mu_L(2 \vee 4) \geq \min\{\mu_L(2), \mu_L(4 \wedge 1)\}$

$\geq \min\{\mu_L(2), \mu_L(1)\}$

$\mu_L(4) \neq \mu_L(2)$

Put c = 1 in equation (2)

$\nu_L(2 \vee 4) \leq \max\{\nu_L(2), \nu_L(4 \vee 1)\}$

$\leq \max\{\nu_L(2), \nu_L(1)\}$

$\nu_L(2) \neq \nu_L(1)$

Put c = 8 in equation (1)

$\mu_L(2 \vee 4) \geq \min\{\mu_L(2), \mu_L(4 \wedge 8)\}$

$\geq \min\{\mu_L(2), \mu_L(4)\}$

$\mu_L(4) = \mu_L(4)$

Put c = 8 in equation (2)

$\nu_L(2 \vee 4) \leq \max\{\nu_L(2), \nu_L(4 \vee 8)\}$

$\leq \max\{\nu_L(2), \nu_L(4)\}$

$\nu_L(2) = \nu_L(2)$

Put c = 16 in equation (1)

$\mu_L(2 \vee 4) \geq \min\{\mu_L(2), \mu_L(4 \wedge 16)\}$

$\geq \min\{\mu_L(2), \mu_L(4)\}$

$\mu_L(4) = \mu_L(4)$

Put c = 16 in equation (2)

$\nu_L(2 \vee 4) \leq \max\{\nu_L(2), \nu_L(4 \vee 16)\}$

$\leq \max\{\nu_L(2), \nu_L(4)\}$

$\nu_L(2) = \nu_L(2)$

$(<\mu_L(2), \nu_L(2)>, <\mu_L(4), \nu_L(4)>)$ is not an IFJSDL.

Follow the remaining examples

IV. INTUITIONISTIC FUZZY MEET SEMI DISTRIBUTIVE LATTICE

4.1. Definition: Let $\langle L, \mu_L, \nu_L \rangle$ is an IFL and is called an intuitionistic fuzzy meet semi distributive lattice if [IFMSDL]

$\mu_L(a \wedge b) = \mu_L(a) \wedge \mu_L(b \vee c)$ and

$\nu_L(a \wedge b) = \nu_L(a) \wedge \nu_L(b \wedge c)$ for

all$(\langle \mu_L(a), \nu_L(a) \rangle, \langle \mu_L(b), \nu_L(b) \rangle, \langle \mu_L(c), \nu_L(c) \rangle) \in$IFL.

4.2. Theorem: Every IFMSDL need not be an IFJSDL

Proof: By an example D_6 is an IFMSDL but not an IFJSDL.

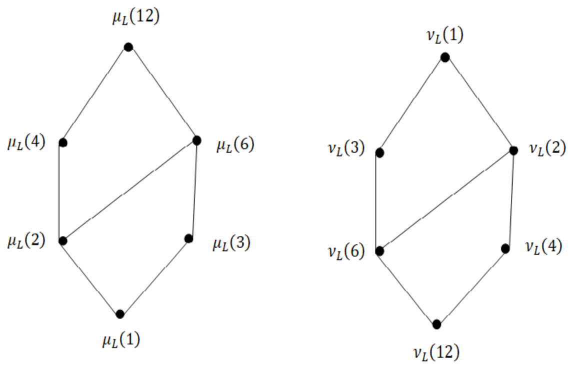

Figure (2) : Intuitionistic fuzzy meet semi distributive lattice of D_6

Then the equations of IFJSDL are

$\mu_L (a \vee b) = \mu_L (a) \vee \mu_L (b \wedge c)$ (1)

$\nu_L (a \vee b) = \nu_L (a) \vee \nu_L (b \vee c)$ (2)

Consider (a, b) = (2,3)

Put c = 1, 4, 6, 12

Put c = 1 in equation (1)

$\mu_L (2\vee 3) \geq \min\{\mu_L (2), \mu_L (3 \wedge 1) \}$

$\geq \min\{\mu_L (2), \mu_L (1) \}$

$\mu_L (6) \neq \mu_L (2)$

Put c = 1 in equation (2)

$\nu_L (2\vee 3) \leq \max\{\nu_L (2), \nu_L (3\vee 1) \}$

$\leq \max\{\nu_L (2), \nu_L (1) \}$

$\nu_L (1) = \nu_L (1)$

Put c = 4 in equation (1)

$\mu_L (2 \vee 3) \geq \min\{\mu_L (2), \mu_L (3 \wedge 4) \}$

$\geq \min\{\mu_L (2), \mu_L (1) \}$

$\mu_L (6) \neq \mu_L (2)$

Put c = 4 in equation (2)

$\nu_L (2 \vee 3) \leq \max\{\nu_L (2), \nu_L (3\vee 4) \}$

$\leq \max\{\nu_L (2),\nu_L (1) \}$

$\nu_L (1) = \nu_L (1)$

Put c = 6 in equation (1)

$\mu_L (2 \vee 3) \geq \min\{\mu_L (2), \mu_L (3\wedge 6) \}$

$\geq \min\{\mu_L (2), \mu_L (3) \}$

$\mu_L (6) = \mu_L (6)$

Put c = 6 in equation (2)

$\nu_L (2 \vee 3) \leq \max\{\nu_L (2), \nu_L (3\vee 6) \}$

$\leq \max\{\nu_L (2), \nu_L (3) \}$

$\nu_L (1) = \nu_L (1)$

Put c = 12 in equation (1)

$\mu_L (2\vee 3) \geq \min\{\mu_L (2), \mu_L (3 \wedge 12) \}$

$\geq \min\{\mu_L (2), \mu_L (3) \}$

$\mu_L (6) = \mu_L (6)$

Put c = 12 in equation (2)

$\nu_L (2 \vee 3) \leq \max\{\nu_L (2),\nu_L (3 \vee 12) \}$

$\leq \max\{\nu_L (2),\nu_L (3) \}$

$\nu_L (1) = \nu_L (1)$

$(< \mu_L (2),\nu_L (2) >, < \mu_L (3), \nu_L (3)>)$ is not an IFJSDL.

Follow the remaining examples

4.3. Definition: Let $\langle L, \mu_L, \nu_L \rangle$ is an IFL and is called an intuitionistic fuzzy distributive lattice [IFDL] if the following conditions are satisfied

$\mu_L (a) \vee \mu_L (b \wedge c) = \mu_L (a \vee b) \wedge \mu_L (a \vee c)$ and

$\nu_L (a) \wedge \nu_L (b \wedge c) = \nu_L (a \wedge b) \wedge \nu_L (a \wedge c)$

for all$(\langle \mu_L (a),\nu_L (a) \rangle,\langle\mu_L (b),\nu_L (b) \rangle,\langle\mu_L (c), \nu_L (c) \rangle) \in$IFL.

4.4. Theorem: Every intuitionistic fuzzy meet semi distributive lattice[IFMSDL] is an IFL and the converse is not true

Proof: Given ⟨L, μ_L, ν_L ⟩ is an IFMSDL

μ_L (a ∧ b) = μ_L (a) ∧ μ_L (b ∨ c) and

ν_L (a ∧ b) = ν_L (a) ∧ ν_L (b ∧ c) for all(⟨ μ_L (a), ν_L (a) ⟩, ⟨μ_L (b), ν_L (b) ⟩, ⟨μ_L (c), ν_L (c) ⟩)∈IFL.

To prove that ⟨L, μ_L,ν_L ⟩ is an IFL

That is to prove μ_L (a ∧ b) = μ_L (a ∧ c) and ν_L (a ∧ b) = ν_L (a ∧ c) for all (⟨ μ_L (a), ν_L (a) ⟩, ⟨μ_L (b), ν_L (b) ⟩, ⟨μ_L (c), ν_L (c) ⟩) ∈IFL.

Then μ_L (a ∧ b) = μ_L (a) ∧ μ_L (b ∨ c)

≥ min{μ_L (a), μ_L (b ∨ c) }

≥ min{μ_L (a), min{μ_L (b), μ_L (c) } }

≥ min{μ_L (a), min{μ_L (c), μ_L (b) } }

≥ min{μ_L (a), μ_L (c ∨ b) }

= μ_L (a) ∧ μ_L (c ∨ b)

= μ_L (a ∧ c)

Also ν_L (a ∧ b) = ν_L (a) ∧ ν_L (b ∧ c)

≤ max{ν_L (a),ν_L (b ∧ c) }

≤ max{ν_L (a), max{ν_L (b),ν_L (c) } }

≤ max{ν_L (a), max{ν_L (c),ν_L (b) } }

≤ max{ν_L (a), ν_L (c ∧ b) }

= ν_L (a) ∧ ν_L (c ∧ b)

= ν_L (a ∧ c)

Hence ⟨L, μ_L, ν_L ⟩ is an IFL.

The converse need not be true, that is every IFL need not be an IFMSDL.

We shall verify it by the following example

Consider an IFL S_8 of the following figure

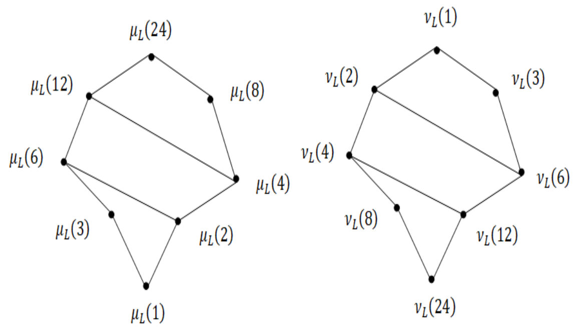

Figure (3). Intuitionistic fuzzy lattice of S8

μ_L (a ∧ b) = μ_L (a) ∧ μ_L (b ∨ c) (1)

ν_L (a ∧ b) = ν_L (a) ∧ ν_L (b ∧ c)(2)

Consider (a, b) = (3,2)

Put c = 4, 6, 8, 12

Put c = 4 in equation (1)

μ_L (3 ∧ 2) ≥ min{μ_L (3), μ_L (2 ∨ 4) }

≥ min{μ_L (3), μ_L (4) }

= μ_L (1)

μ_L (1) = μ_L (1)

Put c = 4 in equation (2)

ν_L (3 ∧ 2) ≤ max{ν_L (3),ν_L (2∧4) }

≤ max{ν_L (3),ν_L (4) }

= ν_L (12)

ν_L (6) ≠ ν_L (12)

Put c = 6 in equation (1)

μ_L (3 ∧ 2) ≥ min{μ_L (3), μ_L (2∨6) }

≥ min{μ_L (3), μ_L (12) }

= μ_L (3)

μ_L (1) = μ_L (3)

Put c = 6 in equation (2)

$\nu_L (3 \wedge 2) \leq \max\{\nu_L (3), \nu_L (2 \wedge 6)\}$

$\leq \max\{\nu_L (3), \nu_L (6)\}$

$= \nu_L (6)$

$\nu_L (6) = \nu_L (6)$

Put c = 8 in equation (1)

$\mu_L (3 \wedge 2) \geq \min\{\mu_L (3), \mu_L (2 \vee 8)\}$

$\geq \min\{\mu_L (3), \mu_L (8)\}$

$= \mu_L (1)$

$\mu_L (1) = \mu_L (1)$

Put c = 8 in equation (2)

$\nu_L (3 \wedge 2) \leq \max\{\nu_L (3), \nu_L (2 \wedge 8)\}$

$\leq \max\{\nu_L (3), \nu_L (8)\}$

$= \nu_L (8)$

$\nu_L (6) \neq \nu_L (8)$

Put c = 12 in equation (1)

$\mu_L (3 \wedge 2) \geq \min\{\mu_L (3), \mu_L (2 \vee 12)\}$

$\geq \min\{\mu_L (3), \mu_L (12)\}$

$= \mu_L (3)$

$\mu_L (1) \neq \mu_L (3)$

Put c = 12 in equation (2)

$\nu_L (3 \wedge 2) \leq \max\{\nu_L (3), \nu_L (2 \wedge 12)\}$

$\leq \max\{\nu_L (3), \nu_L (12)\}$

$= \nu_L (12)$

$\nu_L (6) \neq \nu_L (12)$

$(<\mu_L (3), \nu_L (3)>, <\mu_L (2), \nu_L (2)>)$ is not an IFMSDL.

4.5. Theorem: Intuitionistic fuzzy dual of IFJSDL is an IFMSDL

Proof: Given $\langle L, \mu_L, \nu_L \rangle$ is an IFJSDL

$\mu_L (a \vee b) = \mu_L (a \vee c)$ and $\nu_L (a \vee b) = \nu_L (a \vee c)$

Then $\mu_L (a \vee b) = \mu_L (a) \vee \mu_L (b \wedge c)$ and $\nu_L (a \vee b) = \nu_L (a) \vee \nu_L (b \vee c)$

for all $(\langle \mu_L (a), \nu_L (a) \rangle, \langle \mu_L (b), \nu_L (b) \rangle, \langle \mu_L (c), \nu_L (c) \rangle) \in$IFL

Intuitionistic fuzzy dual of $\mu_L (a \wedge b) = \mu_L (a \wedge c)$ and $\nu_L (a \wedge b) = \nu_L (a \wedge c)$

This implies $\mu_L (a \wedge b) = \mu_L (a) \wedge \mu_L (b \vee c)$ and $\nu_L (a \wedge b) = \nu_L (a) \wedge \nu_L (b \wedge c)$

for all $(\langle \mu L (a), \underline{\nu L} (a) \rangle, \langle \mu L (b), \nu L (b) \rangle, \langle \mu L (c), \nu L (c) \rangle) \in \overline{IFL}$

Therefore $(\overline{IFL})$ is an IFMSDL.

4.6. Theorem: Every intuitionistic fuzzy modular lattice need not be an intuitionistic fuzzy meet semi distributive lattice

Proof: Given $\langle L, \mu_L, \nu_L \rangle$ is an IFML

Then $\langle L, \mu_L, \nu_L \rangle$ contains an intuitionistic fuzzy sublattice isomorphic to M_4

An intuitionistic fuzzy lattice $\langle L, \mu_L, \nu_L \rangle$ is an intuitionistic fuzzy modular lattice if and only if it doesn't contain an intuitionistic fuzzy sublattice isomorphic to N_5.

Assume that an intuitionistic fuzzy lattice $\langle L, \mu_L, \nu_L \rangle$ is an intuitionistic fuzzy modular lattice.

To prove that $\langle L, \mu_L, \nu_L \rangle$ doesn't contain an intuitionistic fuzzy sublattice isomorphic to N_5

Suppose $\langle L, \mu_L, \nu_L \rangle$ contain an intuitionistic fuzzy sublattice isomorphic to N_5

Which implies $\langle L, \mu_L, \nu_L \rangle$ is not an intuitionistic fuzzy modular lattice

This is a contradiction.

Hence ⟨L, μ_L, ν_L ⟩ doesn't contain an intuitionistic fuzzy sublattice isomorphic to N_5

Conversely assume that an intuitionistic fuzzy lattice⟨L, μ_L, ν_L ⟩ doesn't contain an intuitionistic fuzzy sublattice isomorphic to N_5

To prove that ⟨L, μ_L, ν_L ⟩ is an intuitionistic fuzzy modular lattice

Suppose ⟨L, μ_L, ν_L ⟩ is not an intuitionistic fuzzy modular lattice

⟨L, μ_L, ν_L ⟩ contain an intuitionistic fuzzy sublattice isomorphic to N_5

This is a contradiction to our assumption

Which implies ⟨L, μ_L, ν_L ⟩ is an intuitionistic fuzzy modular lattice

Therefore ⟨L, μ_L, ν_L ⟩ is not an intuitionistic fuzzy meet semi distributive lattice (every intuitionistic meet semi distributive lattice is an intuitionistic fuzzy lattice and the converse need not be true)

V. CONCLUSION

In this paper the definition of intuitionistic fuzzy join semi distributive lattice and intuitionistic fuzzy meet semi distributive lattice were given. And also the conditions and characterizations of intuitionistic fuzzy operations of join and meet semi distributive lattices and verified them with examples.

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