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# MHD Flow Past an Impulsively Started Vertical Plate with Variable Mass Diffusion and Rotation Effect

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## ABSTRACT

*This paper investigates unsteady MHD flow past an impulsive started vertical plate with variable mass dispersion and Rotation effects is considered here. The governing equations are associated with the present investigation are solved by the efficient Gelerkin method. The velocity, concentration and skin friction are studied for various parameters like mass Grashof number, Magnetic field parameter, Schmidt number, time and rotation parameter.*

***Keywords**:— Rotation effects, mass diffusion, Finite element method, MHD.*

## I. INTRODUCTION

Examination of MHD flow with heat and mass transfer plays an important in engineering sciences. Some basic applications are cooling of nuclear reactors, liquid metals fluid, and power generation system and streamlined. Murali et.al [1] studied an unsteady magnetohydrodynamic free convective flow past a vertical porous plate. Heat and mass transfer effects on unsteady MHD free convection flow near a moving vertical plate in porous medium was studied by Das and Jana [2]. Chaudhary andArpita[3] has given an exact solution of magnetohydrodynamic convection flow past an accelerated surface embedded in a porous medium. Shankar Goud and Raja Shekar[4] have discussed the finite element solution of viscous dissipative effects on unsteady MHD flow past a parabolic started vertical plate with mass diffusion and variable temperature. Muthucumaraswamy and Ganesh [5] studied an unsteady flow of an incompressible fluid past an impulsively started vertical plate with heat and mass transfer. Unsteady MHD flow past impulsively started vertical plate in porous medium with heat source and chemical reaction analyzed numerically by Rajput and Shareef[6]. Hady et.al [7]studied the MHD free convection flow along a vertical wavy surface with heat generation or absorption effect. Unsteady MHD convection flow of polar fluids past a vertical moving porous plate in a porous medium discussed by Kim [8].Shankar Goud [9] have shown MHD flow past a vertical oscillating plate with radiation and chemical reaction in porous medium- finite difference method. Ibrahim and Makinde [10] have discussed radiation effect on chemically reacting (MHD) boundary layer flow of heat and mass transfer through a porous vertical flat plate. Dufour and Soret effects on unsteady MHD free convection and mass transfer flow past a vertical porous plate in a porous medium studied by Alam et.al[11].Muthucumaraswamy[12] presented the effects of a chemical reaction on a moving isothermal vertical surface with suction. Finite element method application of effects on an unsteady MHD convective heat and mass transfer flow in a semi-infinite vertical moving in a porous medium with heat source and suction has been analyzed by BSGoud and Rajashekar [13].MHD flow past an impulsively started vertical plate with variable temperature and mass diffusion studied by Rajput and Kumar[14]. Finite element study of Soret and radiation effects on mass transfer flow through a highly porous medium with heat generation and chemical reaction has been investigated by Shankar Goud and Raja Shekar[15]. Ali Chamkha [16] studied an unsteady MHD convective heat and mass transfer past a semi-infinite vertical permeable moving plate with heat absorption. Vajravelu and Hadjinicolaou [17] analyzed the heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation. Effect of porosity on unsteady MHD flow past a semi-infinite moving vertical plate with time dependent suction studied by Paras Rama et.al[18].Sattar and Alam [19] studied the thermal diffusion as well as transpiration effects on MHD free convection and mass transfer flow past an accelerated vertical porous plate. Chemical reaction and radiation effects on mixed convection heat and mass transfer over a vertical plate in power-law fluid saturated porous medium were analyzed by Srinivasacharya and Swamy Reddy[20].

The aim of this paper is considering the rotation and radiation effects on MHD flow past an impulsively started vertical plate with variable temperature. Solutions of the governing equations are obtained by efficient Galerkin finite element method. The results are shown with the help of graphsand table.

## II. Mathematical Formulation

Consider a two dimensional unsteady MHD flow of viscous incompressible, an electrically conducting fluid induced by past an impulsively started vertical plate with variable mass diffusion. The fluid and the plate rotate as a rigid body with a uniform angular velocity Ω'about y'axis in the presence of the uniform magnetic field $B_0$ normal to the plate. The plate starts moving with a velocity $u' = u_0$ and its own plane it's assume that an induced magnetic field is negligible. Under the above conditions, the governing equations are as follows

$$\frac{\partial u'}{\partial t'} - 2\Omega' v' = \upsilon \frac{\partial^2 u'}{\partial y'^2} + g\beta^*\left(C' - C'_\infty\right) - \frac{\sigma B_0^2}{\rho} u' \quad (1)$$

$$\frac{\partial v'}{\partial t'} + 2\Omega' u' = \upsilon \frac{\partial^2 v'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} v' \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} \quad (3)$$

The boundary conditions are:

$$\left.\begin{array}{ll} t' \le 0 : u' = 0, C' = C'_\infty & \forall y' \\ t' > 0 : u' = u_0, C' = C'_\infty + \left(C'_w - C'_\infty\right)\frac{u_0^2 t'}{\upsilon}, & at \;\; y' = 0 \\ u' \to 0, \;\; C' \to C'_\infty, & y' \to \infty \end{array}\right\} \quad (4)$$

Where $u_0$ is the velocity of the fluid, $T^*_w$ and $C^*_w$ are the temperature and concentration of the wall respectively, $t^*$ is the time.
Introducing the following non- dimensional quantities,

Where *Gr, Pr, Ha, Ec, l* and *Q* are the thermal Grashof number, Prandtl number, Hartmann number, Eckert number, characteristic length scale and heat absorption parameter respectively.

With the help of non-dimensional quantities, equations (1) and (2) becomes

$$\frac{\partial u}{\partial t} - 2\Omega v = \upsilon\frac{\partial^2 u}{\partial y^2} + GmC - Mu \qquad (6)$$

$$\frac{\partial v}{\partial t} + 2\Omega u = \frac{\partial^2 v}{\partial y^2} - Mv \quad (7)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} \qquad (8)$$

The corresponding nondimensional boundary conditions are

$$\left.\begin{array}{ll} t \le 0 : u = 0,\ C = 0 & \forall y \\ t > 0 : u = 1, C = t & at\ y = 0 \\ u \to 0,\ C \to 0 & as\ y \to \infty \end{array}\right\} \quad (9)$$

## III. Method of Solution

On solving the equations (6)-(8). By applying Galerkin finite element method for equation (6) over the element (e) $(y_j \le y \le y_k)$ is:

$$\int_{y_j}^{y_k} N^{(e)^T}\left[\frac{\partial^2 v^{(e)}}{\partial y^2} - \frac{\partial v^{(e)}}{\partial t} - Mv^{(e)} + R\right] \qquad (10)$$

Where $R = 2\Omega u$ integrating the first term in equation (10) by parts and neglecting the first term we get

$$\left\{\int_{y_j}^{y_k} \frac{\partial N^{(e)^T}}{\partial y}\frac{\partial \theta^{(e)}}{\partial y} + N^{(e)^T}\left(\frac{\partial \theta^{(e)}}{\partial t} + M\theta^{(e)} - R\right)\right\}dy = 0$$

Let $u^{(e)} = N^{(e)}\phi^{(e)}$ = be the linear piecewise approximation solution over the element (e) ( $y_j \le y \le y_k$ ) where

$N^{(e)} = [N_j \quad N_k],\ \phi^{(e)} = [\theta_j \quad \theta_k]^T$ and

$N_j = \frac{y_k - y}{y_k - y_j}, N_k = \frac{y - y_j}{y_k - y_j}$ are the basis functions.

On simplifying we get

$$\frac{1}{l^{(e)}}\begin{bmatrix}1 & -1\\ -1 & 1\end{bmatrix}\begin{bmatrix}\theta_j\\ \theta_k\end{bmatrix} + \frac{l^{(e)}}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}\dot{\theta}_j\\ \dot{\theta}_k\end{bmatrix} + \frac{Ml^{(e)}}{6}\begin{bmatrix}2 & 1\\ 1 & 2\end{bmatrix}\begin{bmatrix}\theta_j\\ \theta_k\end{bmatrix} = \frac{Rl^{(e)}}{2}\begin{bmatrix}1\\ 1\end{bmatrix}$$

where dot are denotes differentiation with respect to y. Assembling the element equations for two consecutive elements $(y_{i-1} \le y \le y_i)$ and $(y_i \le y \le y_{i+1})$ following is obtained:

$$\frac{1}{l^{(e)^2}}\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix}\begin{bmatrix}\theta_{i-1}\\ \theta_i\\ \theta_{i+1}\end{bmatrix} + \frac{1}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}\dot{\theta}_{i-1}\\ \dot{\theta}_i\\ \dot{\theta}_{i+1}\end{bmatrix} - \frac{M}{6}\begin{bmatrix}2 & 1 & 0\\ 1 & 4 & 1\\ 0 & 1 & 2\end{bmatrix}\begin{bmatrix}\theta_{i-1}\\ \theta_i\\ \theta_{i+1}\end{bmatrix} = \frac{R}{2}\begin{bmatrix}1\\ 2\\ 1\end{bmatrix} \qquad (11)$$

Now put row corresponding to the node *i* to zero, from equation (11) the difference schemes with $l^{(e)} = h$ is:

$$\frac{1}{6}\left[\dot{\theta}_{i-1} + 4\dot{\theta}_i + \dot{\theta}_{i+1}\right] + \frac{1}{h^2}\left[-\theta_{i-1} + 2\theta_i - \theta_{i+1}\right] + \frac{M}{6}\left[\theta_{i-1} + 4\theta_i + \theta_{i+1}\right] = R$$

Applying Crank – Nicholson method to the above equation, we get

$$B_1\theta_{i-1}^{j+1} + B_2\theta_i^{j+1} + B_3\theta_{i+1}^{j+1} = B_4\theta_{i-1}^{j} + B_5\theta_i^{j} + B_6\theta_{i+1}^{j} + P^{**} \qquad (12)$$

Where

$B_1 = B_3 = 2 - 6r + kM. \quad B_4 = B_6 = 2 + 6r - kM$

$B_2 = 8 + 12r + 4kM, \quad B_5 = 8 - 12r + 4kM, P^{**} = -12k\Omega u$

Now from equation (7) and (8) following equations are obtained:

$$A_1u_{i-1}^{j+1} + A_2u_i^{j+1} + A_3u_{i+1}^{j+1} = A_4u_{i-1}^{j} + A_5u_i^{j} + A_6u_{i+1}^{j} + P^{*} \quad (13)$$

$$C_1C_{i-1}^{j+1} + C_2C_i^{j+1} + C_3C_{i+1}^{j+1} = C_4C_{i-1}^{j} + C_5C_i^{j} + C_6C_{i+1}^{j} \quad (14)$$

Where

$A_1 = A_3 = 2 - 6r + kM, \quad A_4 = A_6 = 2 + 6r - kM, A_2 = 8 + 12r + 4kM,$

$A_5 = 8 - 12r - 4kM, P^* = 12kP = 12kGmC_i^j + 24k\Omega v_i^j$

$C_1 = C_3 = Sc - 3r, \quad C_4 = C_6 = Sc - +3r, C_2 = 4Sc + 6r,$

$C_5 = 4Sc + 6r.$

Here, r = k / h[2] and h, k are mesh size along the y direction and the time direction respectively. Index i refers to the space, and j refers to the time. In the Equations (7) – (8), taking *i* = 1.n and using boundary conditions (9), the following system of equations are obtained:

$$A_iX_i = B_i \quad i = 1\ldots, n \qquad (15)$$

Where $A_i$'s are matrix of order n and $X_i$, $B_i$'s column matrices having n components. The solutions of above systems of equations are obtained by using the Thomas algorithm for velocity, temperature and concentration. Also the numerical solutions are obtained by executing the MATLAB program with the smaller values of $h$ and $k$. No significant change was observed in $u$, and $\Theta$ then the Galerkin finite element method is stable and convergent. The dimension less skin friction obtained as

$$\tau = \left(\frac{\partial u}{\partial y}\right)_{y=0}$$

## IV. Results and Discussion

The influence of for the different values of physical parameters M, Sc, Gm, Ω and t are presented in figures. Figure 1 and 2 shows that the effect of the magnetic parameter m on the primary and secondary velocity. It is found that the primary and secondary velocity increases as magnetic parameter increases.

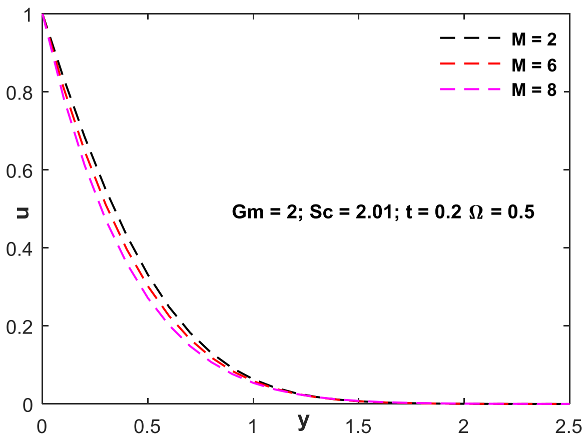


*Figure 1: Primary Velocity Profiles v/s M*

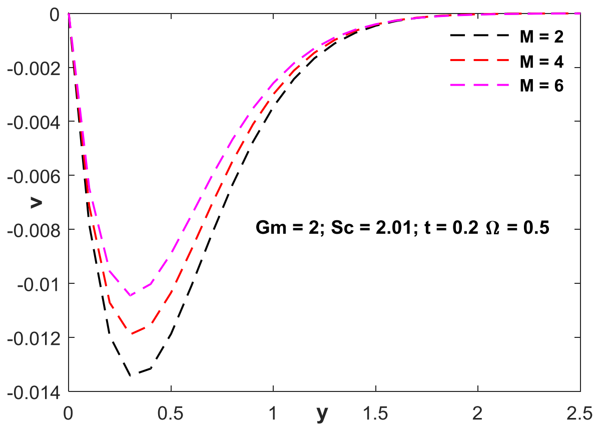


*Figure 2: Secondary Velocity Profiles v/s M*

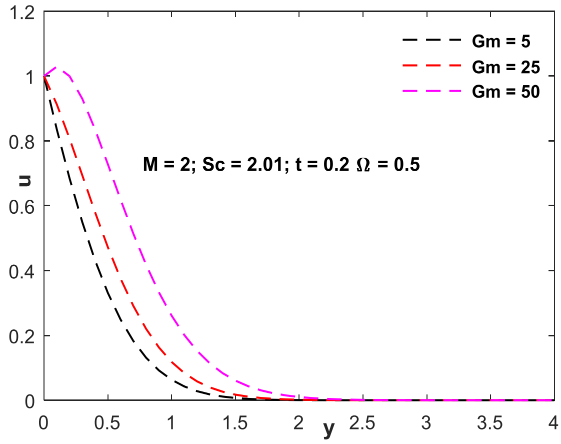


*Figure 3: Primary velocity profiles v/s Gm*

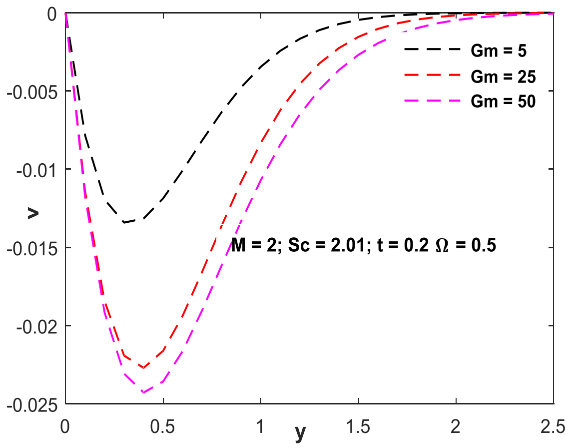


*Figure 4: secondary velocity profiles v/s Gm*

The primary and secondary velocity profilesare plotted in figure 3 and 4, respectively, for different values of the Grashof number (Gm). The result indicates that the primary velocity increases and secondary velocity decreases with an increase of Grashof number.

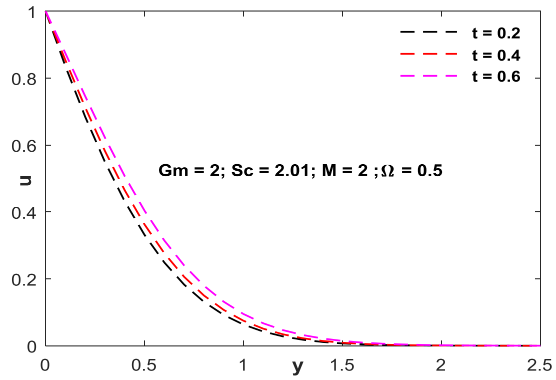


*Figure 5: Primary velocity profiles v/s t*

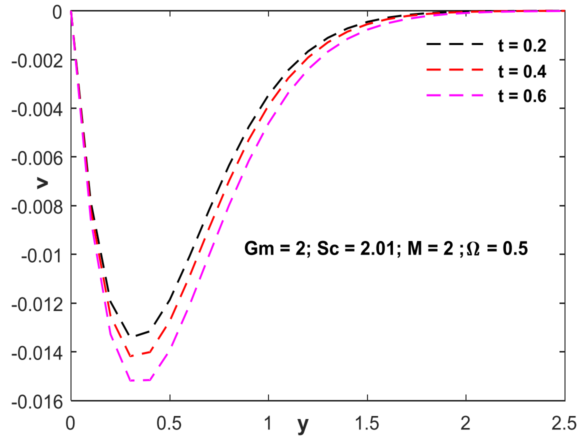


*Figure 6: secondary velocity profiles v/s t*

The effect of time (t) on primary and secondary velocity has been illustrated in figure 5 and 6. It is clear that the primary velocity increases and secondary velocity decreases when increasing the values of time (t).

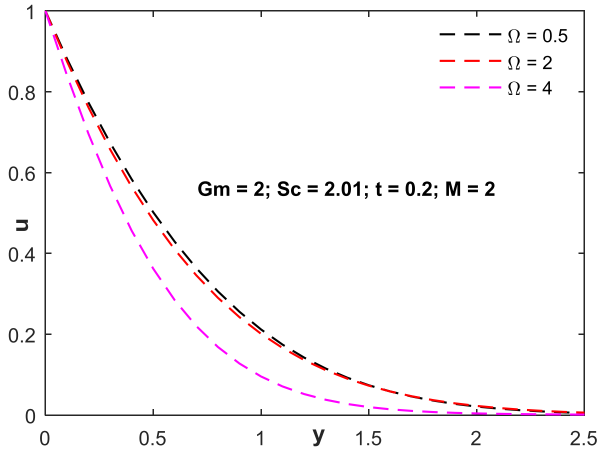


*Figure 5: Primary velocity profiles v/s Ω*

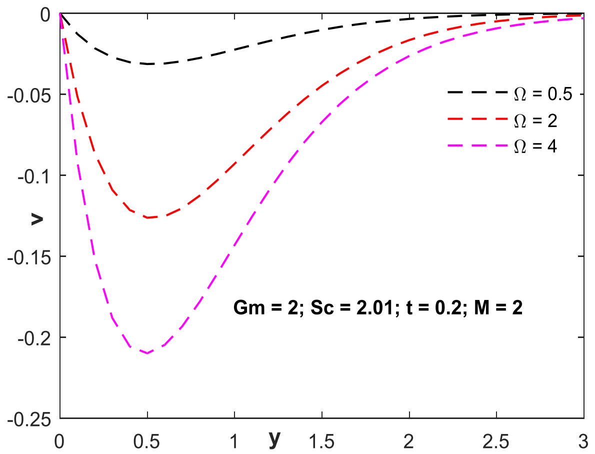


*Figure 6: secondary velocity profiles v/s Ω*

Figure 5 and 6 shows that the effect of the rotation parameter (Ω) on primary and secondary velocity. It is found that the primary and secondary velocity decreases with an increase of rotation parameter (Ω).

## V. Conclusion

In this paper, consider the rotation and radiation effects are considered on MHD flow past an imprudently started vertical plate with variable mass diffusion is studied. Solutions for the model have been proscribed by using Galerkine finite element method.

A couple of completions of the examination are as shown below:

- With increasing the Gm, t the Primary velocity (u) increases and decreases with increase in Ω and M.
- Secondary velocity (v) increases with the augmentation in M, and reduces with increase in GM t and Ω.

| **Nomenclature** | | | |
|---|---|---|---|
| $u', v'$ | Velocity components | $\sigma$ | Electric conductivity |
| $g$ | Acceleration due to gravity | $t'$ | Time |
| $u_0$ | Velocity of the Fluid, | $\rho$ | Density |
| $\Omega'$ | Rotation parameter. | $\upsilon$ | Kinematic viscosity |
| $C_p$ | Specific heat at constant pressure | $B_0$ | External magnetic Field, |
| $C'$ | Concentration in the fluid, | $C'_w$ | Concentration of the Fluid, |
| $C'_\infty$ | Concentration in the fluid Far away from the plate | $D$ | Chemical molecular diffusivity, |
| $u, v$ | Dimensionless velocities | $C$ | Dimensionless concentration |
| $Gm$ | Mass Grashof number | $Sc$ | Schmidt number, |
| $\Omega$ | Dimensionless rotation parameter | $M$ | Magneticfield parameter |

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